MATHEMATICAL MODEL FOR THE ANALYSIS OF FOUR-BAR BEAT-UP MECHANISMS

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Abstract: A mathematical model has been developed for the study of beat-up mechanisms of the four-bar linkage type. The proposed model avoids the drawbacks of the one currently used. It allows accurate determination of the kinematic parameters (magnitude and direction) of an arbitrary point on the beater at each moment of its motion, as well as for different values of the input angular velocity ω and angular acceleration ε. It also allows the determination of the offset factor, if any. The mathematical model applies to any software for engineering calculations, and the final results are obtained with the desired accuracy - approximations and provisionality are not used. MathCAD and AutoCAD were used for the mathematical calculations and graphing constructions (drawing) in this work.

Keywords: kinematic analysis, four-bar linkages, beat-up mechanism

1. Introduction

The beat-up mechanism is one of the main mechanisms of the loom. To fulfill its purpose, the beater must remain at the rear dead center for as long as possible to provide enough time for the weft to be drawn through and then, with one or two striking movements, to compact each thread passed through towards the end of the already woven fabric [1].

In the practice of the weaving industry, beat-up mechanisms are divided into two main types: lever, which can be four-bar, six-bar, or multi-bar, and cam type, also called special mechanisms. Regardless of their type, a various of requirements are imposed on them [2, 3] from both a mechanical and technological point of view. Its kinematic and dynamic characteristics are essential for the proper functioning of the technological process. This necessitates the need for proper research into existing and the synthesis of new beat-up mechanisms.

Due to their simple construction, articulated four-bar beat-up mechanisms are still the most widely used, especially in shuttle and rapier looms. The connection between the connecting rod *AM* and the rocker arm *CM* – point *M* is a special point and is called the sword pin or connecting pin (Fig. 1). Four-bar beat-up mechanisms are classified according to two of their construction features:

- According to the ratio of the lengths of the connecting rod $l = AM$ and the crank $r = OA$, are: with a long connecting rod, when $l/r \geq 6$; with a normal connecting rod, when l/r = 2,5 4 6 and with a short connecting rod, when $l/r < 2{,}5$.
- According to the end positions M_1 and M_2 of the sword pin relative to the axis of rotation *O* of the crank *r*: the mechanism is axial ($e = 0$) if the straight line defined by M_1 and M_2 passes through (crosses) the axis of rotation of the crank and non-axial (sley-eccentricity) ($e \neq 0$) if it does not pass through the axis of rotation point *O*.

A number of parameters of the beat-up mechanisms are being studied to improve their performance. Some of them are focused on the force characteristics of the impact [4], and others on the kinematic and dynamic characteristics of the beat-up mechanisms [2-3, 5-7].

2. Features in determining the kinematic characteristics of axial and non-axial beat-up mechanisms

In axial beat-up mechanisms, the angle of rotation of the main shaft of the machine for moving the beater from the backmost to the frontmost position and the angle of rotation for the reverse movement are equal (identical) (180° each). The movement is simply harmonious; the maximum speed is achieved in the middle of the beater's motion, both in its forward and backward movement.

In the presence of eccentricity, in non-axial beat-up mechanisms, these angles are different. The maximum velocity of the beater, as well as half of the distance, is achieved (accomplished) earlier when moving backwards and later when moving forwards. This way, the beater stays closer to the rearmost position for a longer time and has a higher velocity at the moment of shock. The greater the eccentricity coefficient (non-axiality), the greater the delay. This makes it easier to pass the weft and increases the compaction efficiency. On the other hand, high values of the sley-eccentricity ratio have their disadvantages, such as high acceleration, which decreases at the moment of impact. This leads to a significant increase in the forces acting in the links and pin joints of the kinematic chain of the beat-up mechanism. Vibration and wear increase. In farthermost positions, with a sharp change in the kinematic and dynamic parameters, thread breakage is observed, which leads to a deterioration in the performance of the loom and the quality of the finished product (the fabric).

Some authors [2, 3] propose that the kinematic characteristics of the characteristic point M of the beater be determined (coordinate system O_1xy), assuming that it moves along the chord M_1M_2 (Figure 1).

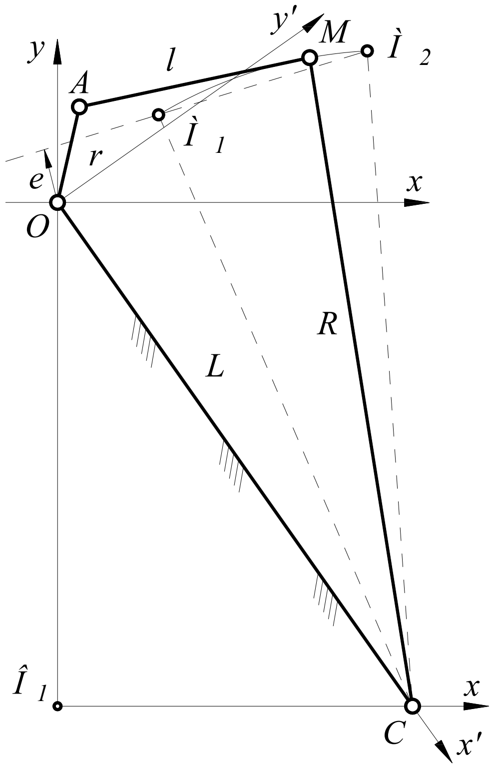


***Figure 1.** Kinematic diagram of a beat-up mechanism*

Under these conditions, the remove (displacement) is:

$$S_M = (r+l) - \left(r\cos\varphi + l\sqrt{1-\frac{r^2}{l^2}\sin^2\varphi} \right) \tag{1}$$

After mathematical processing, acceptance of some approximations, and successive double differentiation, the kinematic characteristics of point M can be determined by:

- for axial beat-up mechanism:

$$S_M = r(1-\cos\omega t) \pm \frac{r^2}{2l}\sin^2\omega t \tag{2}$$

$$V_M = r\omega\left(\sin\omega t \pm \frac{r}{2l}\sin 2\omega t \right) \tag{3}$$

$$a_M^t = r\omega^2\left(\cos\omega t \pm \frac{r}{l}\cos 2\omega t\right) \tag{4}$$

where: the "+" sign is when the beater moves from the frontmost to the rearmost position, i.e., backwards, and the "-" sign is when it moves from the rearmost to the frontmost position, i.e., frontwards.

- for non-axial beat-up mechanism:

$$S_M = q - r\left(\cos\varphi - \frac{r}{2l}\sin^2\varphi - \frac{e}{l}\sin\varphi\right) \tag{5}$$

$$V_M = V_A\left(\sin\varphi + \frac{r}{2l}\sin 2\varphi + \frac{e}{l}\cos\varphi\right) \tag{6}$$

$$a_M^t = \frac{V_A^2}{r}\left(\cos\varphi + \frac{r}{l}\cos 2\varphi - \frac{e}{l}\sin\varphi\right) \tag{7}$$

where: $q = OM_2 - l + \frac{e^2}{2l}$.

In the mathematical model proposed in the scientific literature and used to date, the following disadvantages (drawbacks) have been noted:

1. A preliminary determination of the sley-eccentricity ratio e is required.
2. The assumption that point M moves along the chord M_1M_2 leads to the assumption that the velocity V_M and the tangential acceleration a_M^t are also directed along the chord, which is incorrect. When determining the dynamic characteristics of the beater at the moment of weft compaction, the direction of the speed and acceleration of the characteristic point M is of particular importance.
3. A different mathematical model is used for axial and non-axial beat-up mechanisms.
4. In modern high-speed weaving machines, the use of approximate numerical methods for determining the kinematic characteristics does not yield reliable results, and increasing their accuracy leads to an aggravation of the calculations performed.
5. The mathematical model does not allow for determining the directions of the velocity and acceleration of the characteristic point M (or any other arbitrary point) at any moment of the motion of the beater.
6. Determining the kinematic characteristics of point M for different values of the input angular velocity ω and acceleration ε requires repeated calculations using the presented equations. Solving the problem becomes much more complicated if the input angular velocity ω and acceleration ε are variable functions.

These drawbacks prompted the author to set himself the purpose of developing and proposing a mathematical model for determining the kinematic characteristics (parameters) of the characteristic point M (or other arbitrary point) of the beat-up mechanism, distinguished by simplicity and a certain universality.

3. Mathematical model for the analysis of four-bar beat-up mechanisms

The bear-up mechanism presented in Fig. 1 is a four-bar mechanism of type RRRR with a base L, a crank r, a connecting rod l, and a rocker arm R. The analysis can be conducted in an $Ox'y'$ coordinate system. Determining transfer functions is a known problem from the theory of machines and mechanisms [8-14].

A. Determining coordinates of a point

When studying the beat-up mechanism, it is necessary to determine the coordinates of the sword pin (connecting pin) for a selected position of the crank, as well as its end positions. The dimensions of the units are known, so the coordinates of a desired point can be determined by analytical geometry as the cross point of two circles with centers at two known points (Figure 2).

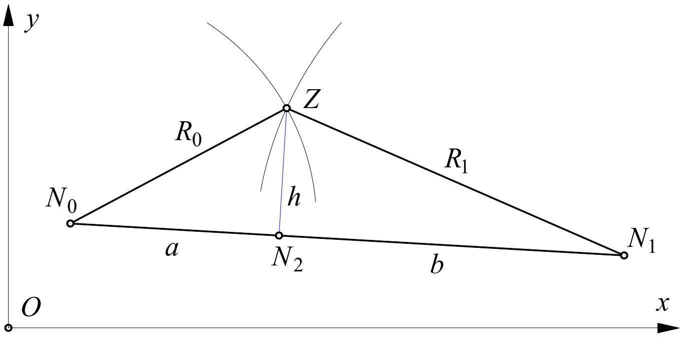


Figure 2. *Determining coordinates of a point*

Suppose that in the coordinate system *Oxy*, the points N_0 and N_1 have coordinates (*x, y*). The lengths R_0 and R_1 are also known. The coordinates of point Z can be determined using the formulas presented in Table 1.

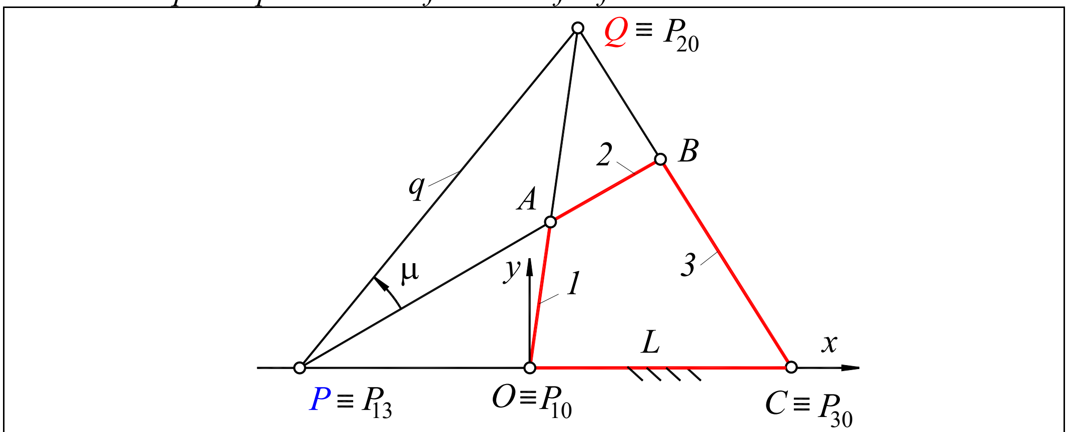
Table 1. *Required parameters for determining the coordinates of a point Z*

Distance dimension $d = a + b$	$d = \sqrt{\left(X_{N_1} - X_{N_0}\right)^2 + \left(Y_{N_1} - Y_{N_0}\right)^2}$
Length of the line segment *a*	$a = \dfrac{R_0^2 - R_1^2 + d^2}{2.d}$
Length of the line segment *h*	$h = \sqrt{R_0^2 - a^2}$
The coordinates of point N_2	$X_{N_2} = X_{N_0} + \dfrac{a.\left(X_{N_1} - X_{N_0}\right)}{d}$ $Y_{N_2} = Y_{N_0} + \dfrac{a.\left(Y_{N_1} - N_{N_0}\right)}{d}$
The coordinates of point *Z*	$X_Z = X_{N_2} \pm \dfrac{h.\left(Y_{N_1} - Y_{N_0}\right)}{d}$ $Y_Z = Y_{N_2} \pm \dfrac{h.\left(X_{N_1} - X_{N_0}\right)}{d}$

B. Transfer functions of a four-bar mechanism

Table 2 shows a methodology for determining the transfer functions of a four-link mechanism by instantaneous center of velocity (ICV) [8].

Table 2. *Required parameters of the transfer functions*



$k_a = y_A / x_A$	Angular coefficient of the position of the crank *OA*
$k_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$	Angular coefficient of the position of the connecting rod *AB*
$k_b = \dfrac{y_B}{x_B - x_C}$	Angular coefficient of the position of the rocker *BC*
$x_Q = \dfrac{k_b . x_C}{k_b - k_a}$ $y_Q = k_a . x_Q$	Coordinates of the absolute ICV point *Q*
$x_P = x_A - \dfrac{y_A}{k_{AB}}$ $y_P = 0$	Coordinates of the relative ICV point *P*
$k_q = \dfrac{y_Q}{x_Q - x_P}$	Angular coefficient of the collineation axis *PQ*
$tg\mu = \dfrac{k_q - k_{AB}}{1 + k_q k_{AB}}$	Angle of the collineation axis *PQ*
$\psi'(\varphi) = \dfrac{x_P}{(x_P - L)}$	First transfer functions
$\psi''(\varphi) = \dfrac{\psi'(1-\psi')}{\tan \mu}$	Second transfer functions
$\omega_4 = \omega_2 . \psi'$ $\varepsilon_4 = \omega_2^2 . \psi'' + \varepsilon_2 . \psi'$	Kinematic parameters

C. End positions of the beater

For the analysis, it is necessary to determine the coordinates of the characteristic point M for a selected crank position OA, as well as for the end positions of the beater – points M_1 and M_2 (Figure 3). Using the methodology presented above, the positions of the two points can be determined as the cross points between two circles. For point M_1, these are k_1 (center point O and radius $r = AM - OA$) and k_0 (center point C and radius $CM = R$), and for point M_2, the circles are k_2 (center point O and radius $r = AM + OA$) and k_0 (center point C and radius $CM = R$ (see also Figure 1)).

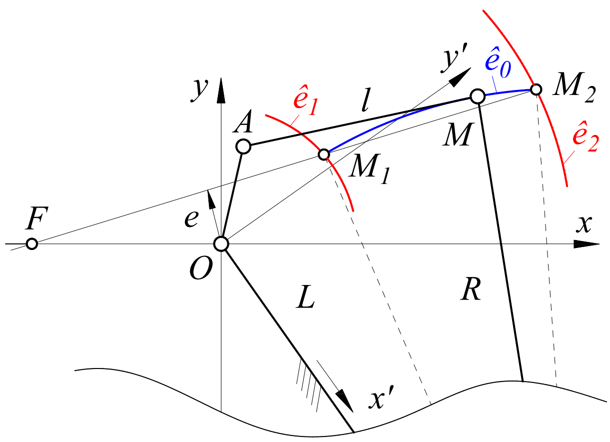


Figure 3. *Determining the position of points M_1 and M_2 and the sley-eccentricity ratio e of the beater*

To determine the disaxiality (sley-eccentricity ratio e), the angular coefficient of the collineation of the straight line M_1M_2, defined by points M_1 and M_2, is used

$$k_{M_1M_2} = \frac{y_{M_2} - y_{M_1}}{x_{M_2} - x_{M_1}} \tag{8}$$

It crosses the coordinate axis O_1x x at point F (the relative ICV) with coordinates:

$$x_F = x_{M_1} - \frac{y_{M_1}}{k_{M_1M_2}} \qquad y_F = 0 \tag{9}$$

The value of the x_F coordinate determines the axial or non-axiality of the mechanism. If:

- $x_F = 0$ - the beater mechanism is axial;
- $x_F > 0$ - the mechanism has an bottom sley-eccentricity;
- $x_F < 0$ - the mechanism has an overhead sley-eccentricity.

The non-axiality of the mechanism can also be determined by the equation:

$$e = x_F \sin\left(arctg\sigma\right) \tag{10}$$

where σ is the angle of the collineation axis FM_1M_2, determined by the formula: $tg\sigma = k_{M_1M_2}$.

Conclusion

A mathematical model for the analysis of a four-bar beat-up mechanism has been developed and presented, which avoids the drawbacks of the one used so far. According to him:

- A preliminary determination of the sley-eccentricity ratio e is not required.
- The analysis of an axial and non-axial mechanism is carried out with the same equations, which are convenient for using a specialized computer program for engineering calculation.
- The final results are obtained with the desired accuracy - approximations and provisionality are not used.
- It allows the calculation of the kinematic characteristics of an arbitrary point on the beater for different values of the input angular velocity ω and angular acceleration ε without the need for additional calculations and without complicating them when functions of ω and ε are variable.
- Allows determination of the directions of velocity and acceleration at any point at any moment of the motion of the beater.

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