Analysis of Cattaneo-Christov Heat flux in Nanofluid, Hybrid and Ternary Hybrid nanofluid flow over rotating Stretchable disk

Nagarjuna Reddy P[1]

[1]Department of Mathematics, SJC Institute of Technology, Chikkaballapur-562 101, India and affiliated to Visveswaraya Technological University, Belagavi-18, India.

Abstract: In this paper, the behavior of nanofluid, hybrid and ternary hybrid nanofluid flow over rotating disk is examined by using Cattaneo - Christov heat flux model . The driving momentum and energy equation are found in cylindrical polar coordinates to show how the flow moves when heat moves. Once these equations are transformed in to non-dimensionless using suitable similarity transformations. The equations were solved by using Spectal-quasi linearization method. Several graphs and table that show how different parameters affect flow and thermal behavior are used to convey results.

1. Introduction

A rotating disk refers to a disk that creates a laminar flow by dragging solution along its surface, and is commonly used for electrochemical measurements or measuring solvent extraction kinetics.

Chunyan Liu et.al [1] has studied the steady flow and heat transfer of Bingham plastic fluid over a rotating disk of finite radius with variable thickness radially in boundary layer. Chenguang Yin et.al [2] has studied flow and heat transfer of Nano fluids over a rotating disk with uniform stretching rate. Tasawar Hayat et.al [3] has Flow of second grade fluid by a rotating disk with heat and mass transfer is discussed. Additional effects of heat generation/absorption are also analysed. Flow is also subjected to homogeneous-heterogeneous reactions. Arsalan Aziz et.al [4] investigated MHD three-dimensional flow of viscous Nano liquid by a rotating disk with heat generation/absorption and slip effects is addressed. Gholinia et.al [5] has studied , the characteristic of the Eyring–Powell Nano fluid flow due to rotating dick with various physical impacts, such as slip flow, magnetic field along with homogeneous–heterogeneous reactions is analysed. Abdul Hafeez et.al [6] has investigated the flow of heat (thermal energy) due to temperature differences. The phenomenon of transfer of heat from one body to another body or in same type bodies occurs because of the temperature difference. As heat transfer is a natural phenomenon, therefore much attentions have developed among the researchers to observe the heat transfer mechanism in the systems. Muhammad Wakeel Ahmad et.al [7] has studied the second grade Nano fluid flow with Cattaneo-Christov heat flux model by a stretching disk is examined in this paper. The Nano fluid flow is characterized with Hall current, Brownian motion and thermophoresis influences. Entropy optimization with nonlinear thermal radiation, Joule heating and heat absorption/generation is also presented. The total entropy profile is enhanced with the Brinkman number, diffusion parameter, magnetic parameter and temperature difference. S.A. Shehzad et.al [8] has studied the Numerical analysis is performed for incompressible Maxwell fluid model flow under the implications of thermophoresis and Cattaneo-Christov theory. Ayele Tulu and Wubshet Ibrahim [9] has studied that the flow due to stretchable rotating disk with the effect of Cattaneo–Christov heat flux model. Both SWCNTs and MWCNTs are considered with ethylene glycol as the base fluid. effects of nanoparticle volume friction, normally applied magnetic field, stretching factor, velocity, and thermal slip factors are examined. Abbasi et.al [10] has investigated elaborates on the flow of viscoelastic Nano fluid consisting of gyro tactic microorganism over a rotating stretching disk along with convective boundary and zero mass flux conditions. Asifa Hassidic et.al [11] has investigated that incompressible hybrid Nano fluid flow over an infinite impermeable rotating disk. The influence of a magnetic field has been added to better examine the fine point of Nano liquid flow. The main purpose of this work is to enhance our understanding of the exhaustion of energy .Yan Zhang et.al [12] has studied that the three-dimensional flow of hybrid Nano fluid owing to a rotating disk of variable thickness in the attendance of modified Fourier law. Hassan Waqas et.al [13] has investigated that the e impacts of non-linear thermal radiations on SWCNT − TiO_2&MWCNT − $CoFe_2O_4$ nanoparticles suspended in water-based hybrid type Nano fluid flow over rotating disk. Adnan et.al [14] has investigated that the thermal transportation in the colloidal suspensions under various scenario becomes an influential research direction due to their extensive applications. M Rahman et.al [15] has studied the impact of uniform suction and magneto hydrodynamics on several Nano fluids over a decelerating rotating disk. It deals with the typical Von Karman flow pumping but in the presence of water based Nano fluids comprising the fraction of the nanoparticle volume of copper, copper oxide, silver, alumina and titanic instead of the regular Newtonian fluid. Sanjay Kumar and Kushal Sharma [16] has studied to the examine the impacts of Stefan blowing on the 3-D Reiner–Rivlin (R–R) fluid flow over a rotating disk moving in the vertical direction. Umar Farooq et.al [17] has investigated the role of iron (III) oxide in heat transfer enhancement over the rotating disk in an axisymmetric flow. Water is utilized as base fluid conveying nano-particle over the revolving axisymmetric flow mechanism.

2. Formulation of the problem

Two dimensional steady, viscous, incompressible laminar flow over rotating stretchable disk has been considered. The flow configuration is given as 2.1 angular velocity of disk represented by Ω_1 , the disk stretchable with stretch quantity b_1 . Magnetic field applied in thedirection of Z-axis. When we move for from the disk the fluid region is called “free stream region”.

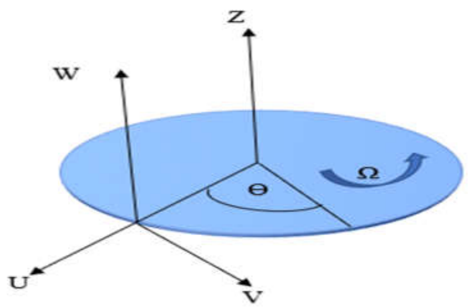

Fig. 1. Geometry and Physical configuration of the problem

Governing equations

$$\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}+\frac{u}{r}=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}=\upsilon_{Tnf}\left(\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial r}+\frac{uv}{r}+w\frac{\partial v}{\partial z}=\upsilon_{Tnf}\left(\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k_{Tnf}}{(\rho c_p)_{Tnf}}\left(\frac{\partial^2 T}{\partial z^2}\right)-\gamma\begin{pmatrix} u^2\frac{\partial^2 T}{\partial r^2}+w^2\frac{\partial^2 T}{\partial z^2}+2uw\frac{\partial^2 T}{\partial z\partial r} \\ +\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)\frac{\partial T}{\partial r} \\ +\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)\frac{\partial T}{\partial z}\end{pmatrix} \tag{4}$$

Associated boundary conditions are

$$\begin{aligned} &u=rb_1, v=r\Omega_1,\ w=0,\ T=T_w \ \text{at}\ z=0 \\ &u=0, v=0,\ \ T=T_\infty \ \text{at}\ z\to\infty \end{aligned} \tag{5}$$

where the physical properties based on hybrid nanofluid are:

kinematic viscosity $\upsilon_{Tnf}=\dfrac{\mu_{Tnf}}{\upsilon_f}$, thermal conductivity $\alpha_{Tnf}=\dfrac{k_{Tnf}}{(\rho C_P)_{Tnf}}$;

dynamic viscosity $\mu_{Tnf}=\dfrac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$,

Thermal diffusivity

$$K_{Tnf}=\left[\frac{K_{np3}+2K_{hnf}-2\phi_3\left(K_{hnf}-K_{np3}\right)}{K_{np3}+2K_{hnf}+\phi_3\left(K_{hnf}-K_{np3}\right)}\right]\left[\frac{K_{np2}+2K_{nf}-2\phi_2\left(K_{nf}-K_{np2}\right)}{K_{np2}+2K_{nf}+\phi_2\left(K_{nf}-K_{np2}\right)}\right]\left[\frac{K_{np1}+2K_f-2\phi_1\left(K_f-K_{np1}\right)}{K_{np1}+2K_f+\phi_1\left(K_f-K_{np1}\right)}\right]K_f$$

density $\rho_{Tnf}=(1-\phi_1)\left\{(1-\phi_2)\left[(1-\phi_3)\rho_f+\phi_3\rho_{np3}\right]+\phi_2\rho_{np2}\right\}+\phi_1\rho_{np1}$

$$(\rho C_p)_{Tnf}=(1-\phi_1)\left\{(1-\phi_2)\left[(1-\phi_3)(\rho C_p)_f+\phi_3(\rho C_p)_{np3}\right]+\phi_2(\rho C_p)_{np2}\right\}+\phi_1(\rho C_p)_{np1}$$

$$\frac{\sigma_{hnf}}{\sigma_{nf}}=\frac{\sigma_{s2}+2\sigma_{nf}-2\phi_2\left(\sigma_{nf}-\sigma_{s2}\right)}{\sigma_{s2}+2\sigma_{nf}+\phi_2\left(\sigma_{nf}-\sigma_{s2}\right)};\frac{\sigma_{nf}}{\sigma_f}=\frac{\sigma_{s1}+2\sigma_f-2\phi_1\left(\sigma_f-\sigma_{s1}\right)}{\sigma_{s1}+2\sigma_f+\phi_1\left(\sigma_f-\sigma_{s1}\right)}$$

Introducing the following transformations to

$$u = r\Omega F'(\eta), v = r\Omega G(\eta), w = -\sqrt{2\Omega \upsilon_f} F(\eta),$$
$$\eta = \sqrt{\frac{2\Omega_1}{\upsilon_f}} z, \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Using (6), eqns. (1) – (5) becomes

$$F''' + C_1 C_2 \left(2FF'' - (F')^2 + G^2\right) = 0 \tag{7}$$

$$G'' + C_1 C_2 \left(FG' - F'G\right) = 0, \tag{8}$$

$$\Theta'' + \frac{C_4}{C_3} \Pr\left[F\Theta' - A\left(F^2\Theta'' + FF'\Theta'\right)\right] = 0 \tag{9}$$

with the boundary conditions

$$F(0) = 0,\ F'(0) = S_p, G(0) = 1, \Theta(0) = 1$$
$$F'(\infty) = 0, G(\infty) = 0, \Theta(\infty) = 0 \tag{10}$$

where $C_1 = \frac{\upsilon_f}{\upsilon_{Tnf}}$, $C_2 = \frac{\rho_{Tnf}}{\rho_f}$ $C_3 = \frac{\alpha_{Tnf}}{\alpha_f}$, $C_4 = \frac{\sigma_{Tnf}}{\sigma_f}$, $A = 2\Omega_1\gamma$

Physical parameters $\Pr = \frac{\upsilon_f}{\alpha_f}$, $S_p = \frac{b}{\Omega}$, $\alpha = 2\ \Omega_1\gamma$

The skin friction coefficients are:

$$C_f = \frac{\tau_{w^*}\big|_{z^*=0}}{\rho_f\left(r^*\Omega\right)^2} \Rightarrow C_f \operatorname{Re}_r^{1/2} = \frac{1}{C_1}\left(\left(F''(0)\right)^2 + \left(G'(0)\right)^2\right)^{1/2} \tag{11}$$

The rate of heat transfer (Nusselt-number):

$$Nu = \frac{-r\, q_w\big|_{z^*=0}}{k_f\left(T_w - T_\infty\right)} \Rightarrow \operatorname{Re}^{-1/2} Nu = -C_3\Theta'(0) \tag{12}$$

Where

$q_w = -k_{hnf}\dfrac{\partial T^*}{\partial z^*}$ is the surface heat flux.

3. Numerical method and Validation

Set of equations [7],[8],[9],[11] are solved by using spectral quasi-linearization method and to get accuracy obtain results comparing with well publish results.

Table . 1 comparison of the results of $F^{|}(0)$ for φ1, φ2, φ3 with the results of Yan Zhang et.al [12]

S_p	$F^{|}(0)$	
	Present result	Yan Zhang et.al [12]
0	-1.1737209	-1.1737
1	-0.948313	-0.9483
2	-0.326244	-0.3262

4. Results and Discussions

The results are discussed relative to the crucial physical parameters, namely the solid volume friction of nanoparticles in ternary hybrid nanofluid consisting of AA7075-AA7072-AI2O3 ,the radial velocity, stretchable parameter S_p thermal relaxation parameter A as presented in fig . 2 -15 . To explain influence of these parameters through behaviour of inertia of flow and non-Fourier heat conductivity in the nanofluid, hybrid nanofluid and ternary hybrid nanofluid.

5. Graphs

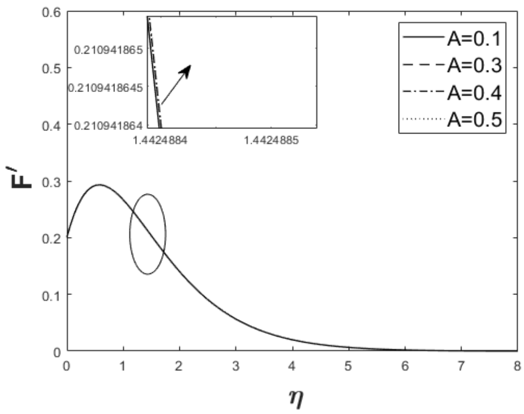

Fig .2: Radial Velocity for different values of A

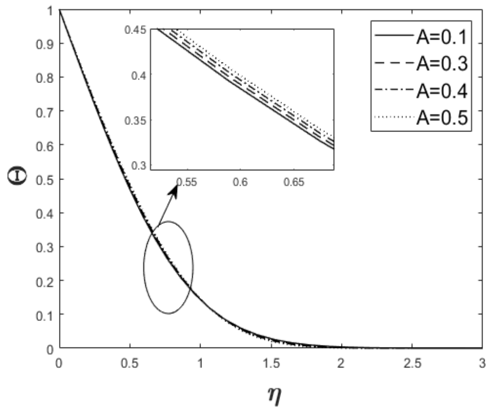

Fig .3: Temperature for different values of A

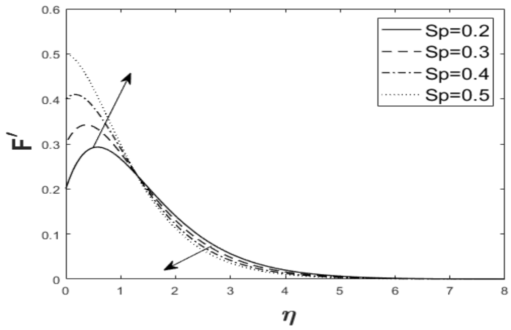

Fig .4: Radial Velocity for different values of Sp

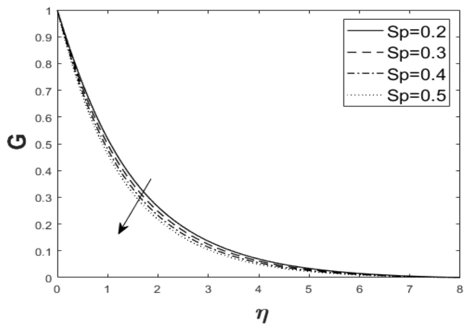

Fig .5: Tangen Velocity for different values of Sp

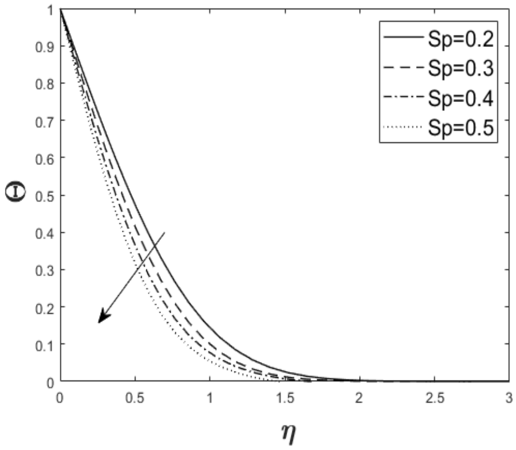

Fig .6: Temperature for different values of Sp

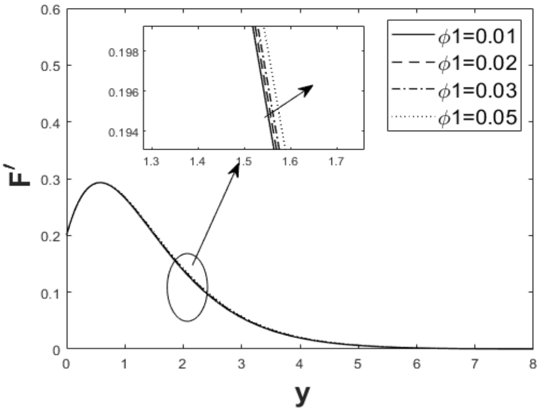

Fig .7: Radial velocity for different values of φ1

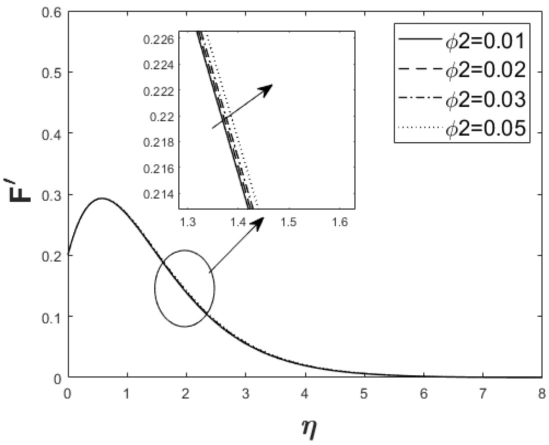

Fig .8: Radial Velocity for different values of φ2

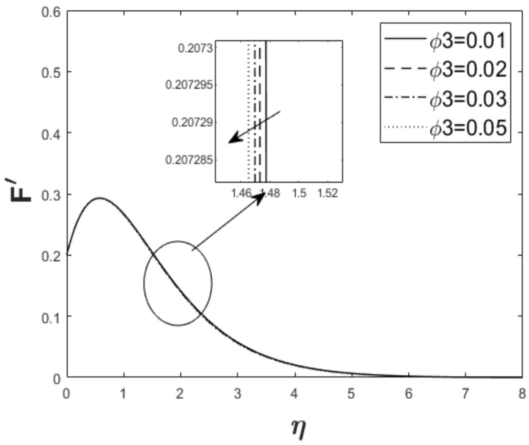

Fig .9: Radial Velocity for different values of φ3

7. Conclusion

The study demonstrates thatinvestigates convective heat transfer in a methanol based ternary hybrid nanofluid containing AA7075, AA7072 and aluminum oxide over a rotating stretchable disk embedded in a porous medium using the Cattaneo–Christov heat flux model. The governing boundary layer equations for momentum and energy with non-Fourier heat conduction are transformed using similarity variables and solved numerically to obtain radial, tangential and axial velocity fields together with the temperature distribution. The effects of thermal relaxation, disk stretchability and the solid volume fractions of the three nanoparticles on the flow and thermal fields, skin friction coefficient and rate of heat transfer are analysed.

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