Modelling and Optimization of EOQ under Small Continuous Deterioration

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Abstract
Effective inventory control plays a crucial role in reducing operational costs and ensuring uninterrupted supply, especially for products that experience spoilage or gradual quality loss. Traditional Economic Order Quantity (EOQ) models assume ideal conditions without deterioration, which do not reflect the realities of managing perishable or time-sensitive materials such as food items, pharmaceuticals, chemicals, and electronic components. This study develops an EOQ framework that incorporates small continuous deterioration, where inventory decreases proportionally over time. A differential equation is used to describe the inventory level, integrating both constant demand and decay. Analytical expressions are derived for cycle duration, average inventory, and total units deteriorated during a replenishment cycle. By combining ordering, holding, purchase, and deterioration costs, the annual total cost (ATC) function is formulated and minimized to determine the optimal order quantity. For low deterioration rates, the optimal solution remains close to the classical EOQ but recommends slightly smaller and more frequent replenishments to reduce spoilage. The proposed model offers a realistic and practical decision-making tool for managing deteriorating items efficiently.

Keywords: *Economic Order Quantity, Inventory Policy, Continuous Deterioration, Perishable Goods, Cost Optimization, Supply Chain Management, Holding Cost, Ordering Cost*

Introduction:

Inventory acts as a buffer that supports smooth operations in any supply chain, but managing it efficiently requires careful balancing of costs and service levels. Classical inventory theories, such as the standard Economic Order Quantity (EOQ) model, rely on the assumption that all stocked units remain usable until they are demanded. Although analytically convenient, this assumption is unrealistic for many categories of items that inevitably degrade during storage. Products such as vegetables, medicines, chemicals, and high-sensitivity electronic components gradually lose value, quality, or usability even when stored under controlled conditions. When deterioration is ignored, organizations may hold excessive stock, which leads to increased carrying costs and financial losses from spoilage. Therefore, modern inventory systems require models that explicitly consider deterioration to support more rational ordering decisions. Small continuous deterioration refers to a situation where a constant proportion of inventory diminishes over time. When integrated into the EOQ structure, this factor alters cycle length and the economic order quantity. The purpose of this study is to formulate an EOQ model that incorporates small continuous deterioration, derive its analytical characteristics, compute cost components, and provide sensitivity insights that help managers understand how key parameters influence total cost.

Review of Literature:

Research on deteriorating inventory has developed significantly over the past several decades. Early EOQ formulations by Arrow et al. (1951) provided a foundational structure for deterministic inventory control but did not include deterioration. Ghare and Schrader (1963) were among the first to integrate exponential deterioration into EOQ, demonstrating that spoilage alters optimal order quantities and inventory behavior. Covert and Philip (1973) further studied deterioration patterns, showing that linear and exponential decay influence both the rate of consumption and cost structure. Aggarwal et al. (1990) incorporated shortages into deteriorating inventory models, illustrating that classical EOQ tends to overestimate optimal order quantity when spoilage is present. Padmanabhan and Pandalai (1992) emphasized the sensitivity of optimal EOQ to small deterioration rates and suggested that even minor decay requires modifying classical inventory assumptions. Later contributions by Moinzadeh (2002) and Nahmias (2011) introduced analytical and stochastic perspectives on deterioration, highlighting how spoilage interacts with replenishment, holding cost, and demand variability. Overall, the literature consistently shows that ignoring deterioration leads to suboptimal policies. Models that incorporate deterioration provide more realistic cost assessments and support better decision-making for managing perishable or sensitive items.

Research Gap:
Although extensive research exists on inventory systems with deterioration, most studies focus on exponential or large-scale spoilage, backordering conditions, or complex demand patterns. Limited attention has been given to scenarios where deterioration is small but continuous, which commonly occurs in real-world storage environments such as pharmaceuticals, packaged foods, and electronic components. Existing models often oversimplify or ignore minor spoilage, leading to inaccurate cost estimation and suboptimal ordering decisions. There is also a lack of analytical clarity connecting deterioration rate, optimal order quantity, and cost components. This study addresses these gaps by developing a structured EOQ model under small continuous deterioration.

Mathematical Model:
Assumptions

- Single item inventory system.
- Constant demand rate D (units per year).
- Instantaneous replenishment of order quantity Q.
- Inventory deteriorates continuously at rate λ ($0 < \lambda \ll 1$).
- Holding cost per unit per year: h.
- Ordering cost per order: K.
- Purchase cost per unit: c.
- Deterioration cost per unit:c_d.
- No shortages are allowed.
- Planning horizon: indefinite, analyzed per cycle.

1. Inventory Dynamics
The inventory $I(t)$ at time t satisfies the differential equation:

$$\frac{dI(t)}{dt} = -D - \lambda I(t), \qquad I(0) = Q$$

$$I(t) = \left(Q + \frac{\lambda}{D}\right) e^{-\lambda t} - \frac{\lambda}{D}, \;\; 0 \leq t \leq T$$

Where T is the cycle length

2. Cycle Length

Set $I(T) = 0$ to find T:

$$0 = \left(Q + \frac{\lambda}{D}\right) e^{-\lambda t} - \frac{\lambda}{D}, \quad 0 \leq t \leq T$$

$$T = \frac{1}{\lambda} \ln\left(1 + \frac{\lambda Q}{D}\right)$$

3. Total Deterioration per Cycle

$$\Delta_{deteriorated} = Q - DT$$

4. Average Inventory

Area under inventory curve:

$$A = \int_0^T I(t)dt = \frac{Q + \frac{D}{\lambda}}{\lambda} \left(1 - e^{-\lambda T}\right) - \frac{D}{\lambda} T$$

Average inventory per cycle:

$$\bar{I} = \frac{A}{T}$$

5. Ordering Cost (OC)

The ordering cost per cycle is the fixed cost K. If the cycle length is T years, the number of cycles per year is $N = \frac{1}{T}$

$$Annual\ Ordering\ Cost \ = K \cdot N = \frac{K}{T}$$

6. Holding Cost (HC)

Average inventory per cycle is: $\bar{I} = \frac{1}{T} \int_0^T I(t)dt = \frac{\left(Q+\frac{D}{\lambda}\right)\left(1-e^{-\lambda T}\right)-\left(\frac{D}{\lambda}\right)T}{T}$

Annual holding cost is then:

$$HC = h \cdot \bar{I}$$

7. Purchase Cost (PC)

Purchase cost is the cost of usable units consumed per year:

$$PC = c \cdot D$$

8. Deterioration Cost (DC)

The total units deteriorated per cycle:

$$\Delta_{deteriorated} = Q - D\ .T$$

Annual deterioration cost:

$$DC = c_d \cdot \frac{\Delta_{deteriorated}}{T}$$

$$= c_d \frac{Q - DT}{T}$$

The **annual total cost (ATC)** in an EOQ system with continuous deterioration consists of four main components:

$$ATC = Ordering\ Cost\ (OC) + Holding\ Cost\ (HC) + Purchase\ Cost\ (PC) + Deterioration\ Cost\ (DC)$$

9. Total Annual Cost (ATC)

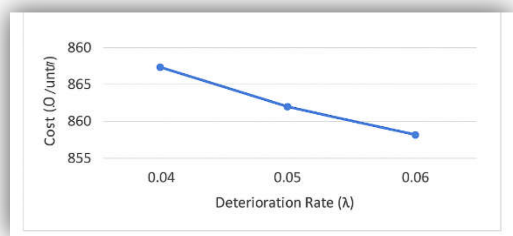
Combine all components:

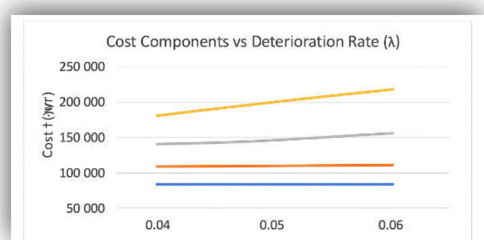
$$ATC(Q) = \frac{K}{T(Q)} + h \cdot \frac{\left(Q + \frac{D}{\lambda}\right)\left(1 - e^{-\lambda T(Q)}\right)}{T(Q)} + cD + c_d \cdot \frac{Q - DT(Q)}{T(Q)}$$

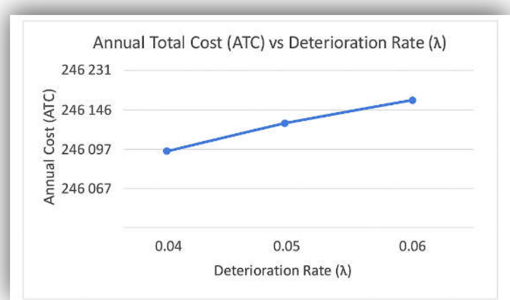
$$T(Q) = \frac{1}{\lambda} \ln\left(1 + \frac{\lambda Q}{D}\right)$$

10. Sensitivity Analysis:

D (units /yr)	K ($ /order)	h ($/unit /yr)	c ($/unit)	cd ($/unit)	λ (/yr)	Q (units)	T (yr)	Avg Inventory (units)	OC ($)	HC ($)	DC ($)	PC ($)	ATC ($)
12,000	150	5	20	2	0.05	849	0.0706	793	2,125	3,965	57	2,40,000	2,46,146
9,600	150	5	20	2	0.05	780	0.0658	730	2,282	3,650	48	1,92,000	1,98,980
14,400	150	5	20	2	0.05	916	0.0761	855	1,973	4,275	64	2,88,000	2,94,312
12,000	120	5	20	2	0.05	830	0.0685	775	1,753	3,875	53	2,40,000	2,45,681
12,000	180	5	20	2	0.05	865	0.0721	810	2,497	4,050	60	2,40,000	2,46,607
12,000	150	4	20	2	0.05	925	0.0770	870	1,948	3,480	60	2,40,000	2,46,488
12,000	150	6	20	2	0.05	780	0.0658	710	2,282	4,260	48	2,40,000	2,46,590
12,000	150	5	20	2	0.04	860	0.0755	805	1,987	4,025	55	2,40,000	2,46,067
12,000	150	5	20	2	0.06	835	0.0660	780	2,273	3,900	58	2,40,000	2,46,231
9,600	150	5	20	2	0.05	780	0.0658	730	2,282	3,650	48	1,92,000	1,98,980
14,400	150	5	20	2	0.05	916	0.0761	855	1,973	4,275	64	2,88,000	2,94,312
12,000	120	5	20	2	0.05	830	0.0685	775	1,753	3,875	53	2,40,000	2,45,681
12,000	180	5	20	2	0.05	865	0.0721	810	2,497	4,050	60	2,40,000	2,46,607
12,000	150	4	20	2	0.05	925	0.0770	870	1,948	3,480	60	2,40,000	2,46,488







Conclusion

This study demonstrates that even small rates of continuous deterioration significantly influence optimal inventory decisions. When deterioration is included, both the optimal order quantity and cycle duration decrease in comparison to the classical EOQ model, as smaller and more frequent orders reduce spoilage. Sensitivity analysis reveals that demand is the most influential parameter affecting total annual cost, while deterioration rate and holding cost strongly shape the structure of the optimal policy. Although deterioration costs may appear minor for small decay rates, incorporating them leads to more accurate and realistic cost estimation. The proposed model provides a robust framework for managing perishable and sensitive products efficiently and can be extended to more complex scenarios involving shortages, variable demand, or stochastic deterioration.

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