A Systematic Review of Attribute and Variable Control Charts in Manufacturing Processes: A Comparative Study of np, p, c, and u Charts with X̄–R and S Charts Across Industries

Ms. Kalaga V N S Anjanee Gayatri, Assistant Professor, Department of Statistics St.Ann's College for Women, Mehdipatnam, Hyderabad,Telangana, India.

+91 9515697414, ORCID:0009-0000-3934-5012

Ms. M. Monika Sai, Assistant Professor, Department of Statistics,

St.Ann's College for Women, Mehdipatnam, Hyderabad,Telangana, India.

+91 8501801452, ORCID:0009-0006-4477-8801

Abstract

This systematic review examines the comparative use of attribute and variable control charts in manufacturing processes, with a focus on np, p, c, and u charts versus X̄–R and S charts across various industries. The primary purpose of this study is to analyze how these two categories of control charts differ in application, sensitivity, and effectiveness for process monitoring and quality improvement. A comprehensive literature-based synthesis was conducted using peer-reviewed articles from Scopus, ScienceDirect, and SpringerLink databases published between 2000 and 2025. Studies were selected based on relevance to statistical process control (SPC) and their documented applications in sectors such as automotive, pharmaceutical, electronics, and textile manufacturing. The findings reveal that attribute control charts are highly effective for monitoring discrete data such as defect counts and proportions, providing simplicity and ease of implementation. In contrast, variable control charts are more suitable for continuous process data, offering greater precision and sensitivity in detecting small process variations. Comparative evidence suggests that combining both chart types can enhance process capability analysis in complex production systems. Emerging trends highlight a growing shift toward automation, integration of multivariate control charts, and the adoption of artificial intelligence (AI)-driven SPC tools for real-time monitoring and predictive quality management. The review concludes by emphasizing the need for hybrid and intelligent control systems to meet the evolving demands of Industry 4.0 manufacturing environments.

Keywords: Statistical Process Control (SPC); Attribute Control Charts; Variable Control Charts; Manufacturing Processes; Quality Control; Industry 4.0; AI-driven SPC

1.Introduction

1.1 Background

Quality control has long been recognized as the cornerstone of manufacturing excellence. It ensures that processes consistently produce outputs that meet customer requirements, reduce waste, and enhance productivity. Among the various quality control techniques, *Statistical Process Control (SPC)* stands as one of the most influential and enduring methodologies for achieving continuous improvement. Developed in the 1920s by Walter A. Shewhart at Bell Laboratories, SPC introduced the concept of using statistical techniques to monitor and control manufacturing processes. Shewhart's pioneering work laid the foundation for distinguishing between *common causes* and *special causes* of variation, thus enabling manufacturers to detect process instability and take corrective actions before defects occur. *(Shewhart, 1931; Malindžáková, Čulková, & Trpčevská, 2023)*

Over the decades, SPC has evolved from manual chart plotting to highly automated, computer-assisted systems capable of real-time data analysis. Its central goal remains the same—to maintain process stability and ensure product quality through systematic observation and data-driven decision-making. Control charts, the visual tools of SPC, serve as early warning systems that help identify deviations from expected performance. By distinguishing random (natural) variability from assignable (abnormal) causes, these charts enable organizations to sustain high-quality production, minimize rework, and improve overall process capability.

In the context of modern manufacturing, SPC plays an increasingly strategic role. Industries such as automotive, pharmaceutical, electronics, and textiles rely on SPC to comply with stringent quality standards, optimize resources, and ensure consistency in global supply chains. As manufacturing transitions toward *Industry 4.0*, integrating data analytics, cyber-physical systems, and artificial intelligence (AI), the application of SPC tools has become more sophisticated and dynamic, supporting predictive quality management and autonomous decision-making.

1.2 Problem Context

Despite the widespread use of SPC, one critical area that requires deeper understanding is the distinction and comparative applicability of *attribute* and *variable control charts*. Attribute data represent qualitative characteristics that are count-based—such as the number of defective units or the proportion of defects in a sample. Charts like *np, p, c,* and *u* are used for monitoring such discrete outcomes. These charts are simple to construct and widely used when measurements are not feasible or practical.

On the other hand, variable data are quantitative and measurement-based, capturing continuous characteristics such as diameter, weight, or temperature. $\bar{X}$–R and S charts are common tools for such data, offering greater sensitivity to small shifts in process mean or variability. However, implementing variable charts requires precise measurement systems and robust data collection mechanisms.

The rapid evolution of manufacturing environments, characterized by automation, sensor integration, and real-time analytics, has created a growing need to systematically compare these two categories of control charts. As industries increasingly adopt smart manufacturing practices, understanding the contexts in which each chart performs optimally—and how they can be integrated—is vital. Moreover, the traditional static nature of control charts is being challenged by the dynamic, high-velocity data streams of modern production systems. Consequently, revisiting their comparative effectiveness under contemporary conditions is both timely and essential.

1.3 Literature Review

Historical foundations and scope of SPC

Statistical Process Control (SPC) originated with Walter A. Shewhart's work in the 1920s–1930s and the development of control charts to separate common-cause from special-cause variation (Shewhart, 1931). Subsequent quality thinkers (e.g., Deming, Juran) and textbook expositions (Montgomery) consolidated SPC as an indispensable statistical framework for monitoring manufacturing processes, performing capability analysis, and supporting continuous improvement programs. The Shewhart ±3σ control-limit principle and the concepts of in-control vs out-of-control remain the conceptual backbone of modern SQC practice.

Attribute control charts: principles and recent developments

Attribute (count-based) charts — principally the np, p, c, and u charts — are used where inspection yields qualitative outcomes (defective/nondefective) or counts of defects per unit. Classical treatments (binomial model for p/np charts, Poisson model for c/u charts) continue to underpin their design and interpretation; their simplicity, low measurement cost, and applicability to high-volume inspection explain their persistent industrial use (Woodall, 2000; NIST/ITL handbook). Recent work has focused on robustness and automation:

• Automated defect detection: The integration of machine-vision systems and automated defect counting pipelines has renewed interest in attribute SPC because vision systems feed continuous streams of count data directly into p/c/u charts, enabling near-real-time monitoring (Zhang et al., 2024).

• Sampling and improved sensitivity: Modifications to sampling plans and dependent-state sampling have been investigated to increase the sensitivity of attribute charts while preserving low inspection cost (Aslam, Khan & Albassam, 2019).

• Variable parameter and time-varying Poisson models: Contemporary studies (e.g., Sałaciński et al., 2023) examine how parameter variability (e.g., changing defect opportunities) affects control-limit calculation for Poisson-based charts and propose generalized control rules.

Variable control charts: precision measurement and extensions

Variable charts ($\bar{X}$–R; $\bar{X}$–S) monitor continuous quality characteristics and are more sensitive to small shifts in process mean or dispersion. They are widely applied in machining, precision assembly, and process industries where accurate measurements are available. Key developments include:

• Adaptive limits and time-truncated charts: Research into adaptive or time-truncated control limits aims to maintain sensitivity while reducing false alarms in non-stationary processes (Kumar & Joshi, 2023). Such approaches are particularly relevant when process behavior evolves over production runs.

• Integration with capability analysis: Variable charts are often used together with capability indices (Cp, Cpk) to quantify how well a controlled process meets specifications (Montgomery, 2019).

• Real-time sensor integration: As measurement data become available from IoT sensors and CNC systems, $\bar{X}$–R and S charts are being implemented in streaming architectures for continuous monitoring and automated alerts (Chen et al., 2024).

Comparative and hybrid approaches: attribute vs variable charts

Comparative studies consistently report the complementary strengths of attribute and variable charts: attribute charts are low-cost and practical for defect monitoring, while variable charts provide superior early detection of small process drifts. Empirical industry studies (e.g., automotive and electronics case studies) show improved detection and decision support when both chart types are used together—attribute charts for pass/fail metrics and variable charts for dimensional control (Bersimis & Psarakis, 2023; Castagliola et al., 2024). Nonetheless, standardized methods for fusing attribute and variable information (e.g., joint alarm rules, composite indices) remain underdeveloped.

SPC in Industry 4.0: multivariate methods, AI, and real-time control

The digitalization of manufacturing has driven several important research directions:

• Multivariate SPC: Hotelling's T^2, MEWMA, and PCA-based SPC address correlated multiple quality characteristics; these methods are essential where quality is multi-dimensional (Bersimis et al., 2023).

• Machine learning and predictive SPC: ML techniques (supervised classifiers, anomaly detectors, and deep learning) are being investigated for predictive quality monitoring that anticipates out-of-control conditions before they manifest on conventional charts (Garg et al., 2025; Zhou et al., 2024).

• Cyber-physical integration: Linking SPC to IIoT platforms and digital twins enables closed-loop responses (automated alarms, maintenance scheduling), but raises challenges related to data reliability, latency, and interpretability.

1.4 Objectives

This paper aims to conduct a *systematic review* of attribute and variable control charts in manufacturing processes, focusing on their comparative strengths, weaknesses, and industrial applications. Specifically, the objectives are:

1. To compare the characteristics and statistical foundations of attribute and variable control charts, emphasizing their suitability for different types of manufacturing data.
2. To evaluate the effectiveness, applicability, and limitations of np, p, c, and u charts in comparison with $\bar{X}$–R and S charts across various industrial domains such as automotive, pharmaceutical, electronics, and textile sectors.
3. To identify research and practical gaps in the existing literature concerning hybrid, automated, or AI-driven SPC systems.
4. To propose future directions for integrating traditional SPC tools with emerging technologies such as machine learning, IoT, and real-time data analytics to enhance predictive quality control.

2. Theoretical Foundation (SQC Overview)

This section presents the theoretical foundation of Statistical Quality Control (SQC) with a particular focus on Statistical Process Control (SPC) and the classification of control charts. SQC forms the backbone of modern quality assurance systems, ensuring that manufacturing and service processes operate efficiently, consistently, and within acceptable limits. SPC, as a subset of SQC, utilizes statistical tools to monitor and control variability in production processes, thereby enhancing reliability, product uniformity, and customer satisfaction.

2.1 Concept of Statistical Process Control (SPC)

2.1.1 Definition and Goals of SPC

Statistical Process Control (SPC) is a methodological approach used to monitor, control, and improve a process through statistical techniques. SPC aims to ensure that processes remain stable and capable of producing output that meets specifications. The primary goal of SPC is not merely to inspect quality after production but to prevent defects by identifying and controlling sources of variation during the process itself.

In essence, SPC emphasizes process stability and continuous improvement. By analyzing process data, SPC helps organizations distinguish between normal, expected variability and unusual, assignable causes that require corrective action. This proactive approach reduces waste, rework, and inspection costs while ensuring consistent quality output. SPC tools, particularly control charts, provide visual insights into process behavior over time and serve as a scientific foundation for decision-making in quality management.

2.1.2 Common Cause vs. Special Cause Variation

A fundamental principle of SPC is understanding the nature of process variability. Every process exhibits variation, but not all variation is of the same kind. **Shewhart** classified variability into two categories:

1. **Common Cause Variation (Natural Variation):**
    - These are inherent fluctuations present in any process due to random factors such as minor environmental changes, machine wear, or human differences.
    - They are stable, predictable, and part of the process's normal operation.
    - A process influenced only by common causes is said to be **in control** and stable over time.
2. **Special Cause Variation (Assignable Variation):**
    - These variations occur due to identifiable, external, or unexpected factors that disturb process stability—such as a broken tool, equipment malfunction, wrong material input, or operator error.
    - Special causes indicate that the process is **out of control** and requires immediate investigation and corrective action.

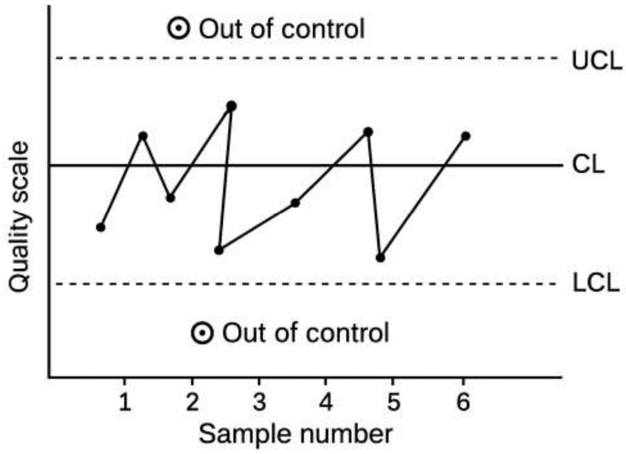
Understanding the distinction between these two types of variation is vital in SPC because it determines whether process changes are necessary. Corrective measures for common cause variation typically involve process redesign or improvement, whereas special causes require immediate root-cause analysis and removal of the disturbance.

2.1.3 Importance of Control Limits (±3σ Concept)

Control charts are the most powerful and widely used SPC tools because they graphically display data over time and indicate whether a process is operating within statistically acceptable limits. These limits are established using the concept of standard deviation (σ), which measures the dispersion of data around the process mean.

Control limits are not the same as specification limits; they are statistically derived boundaries based on process data. The three key horizontal lines on a control chart are:

- **Center Line (CL):** Represents the process average or expected value.
- **Upper Control Limit (UCL):** Located at +3σ from the mean.
- **Lower Control Limit (LCL):** Located at –3σ from the mean.



Mathematically, the control limits are defined as:

UCL = Mean + 3σ
LCL = Mean – 3σ

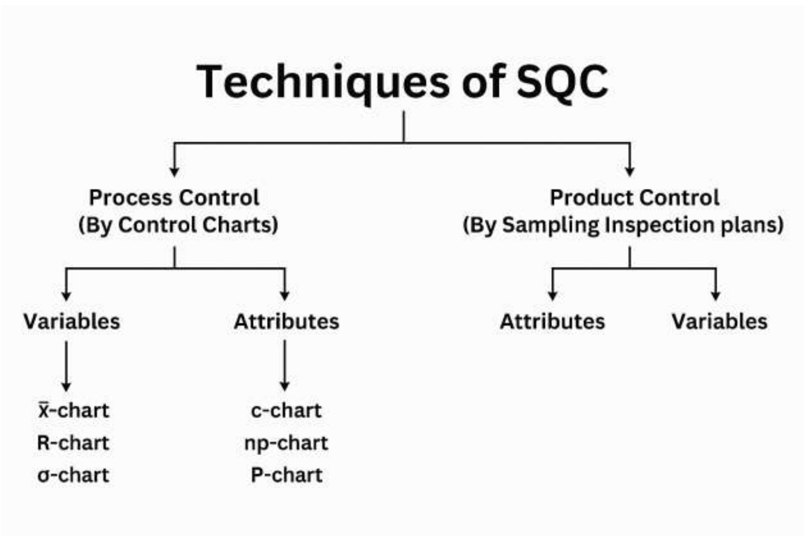
The ±3σ range captures approximately **99.73%** of all data points under a normal distribution, implying that any observation outside these limits is statistically unusual and likely due to a special cause.

If points fall within control limits but display non-random patterns (e.g., trends, cycles, or systematic shifts), the process may still be unstable, requiring further investigation. Thus, control limits serve as early warning indicators, helping quality practitioners detect potential problems before they result in defective output.

SPC's reliance on control limits allows for data-driven decision-making, reducing subjectivity and ensuring process corrections are made only when justified by statistical evidence.

2.2 Classification of Control Charts

Control charts are the core instruments of SPC. They are designed to track process behavior over time and detect variations that may signify process instability. Broadly, control charts are classified into two main types based on the nature of data being monitored: attribute control charts and variable control charts.



2.2.1 Attribute Control Charts

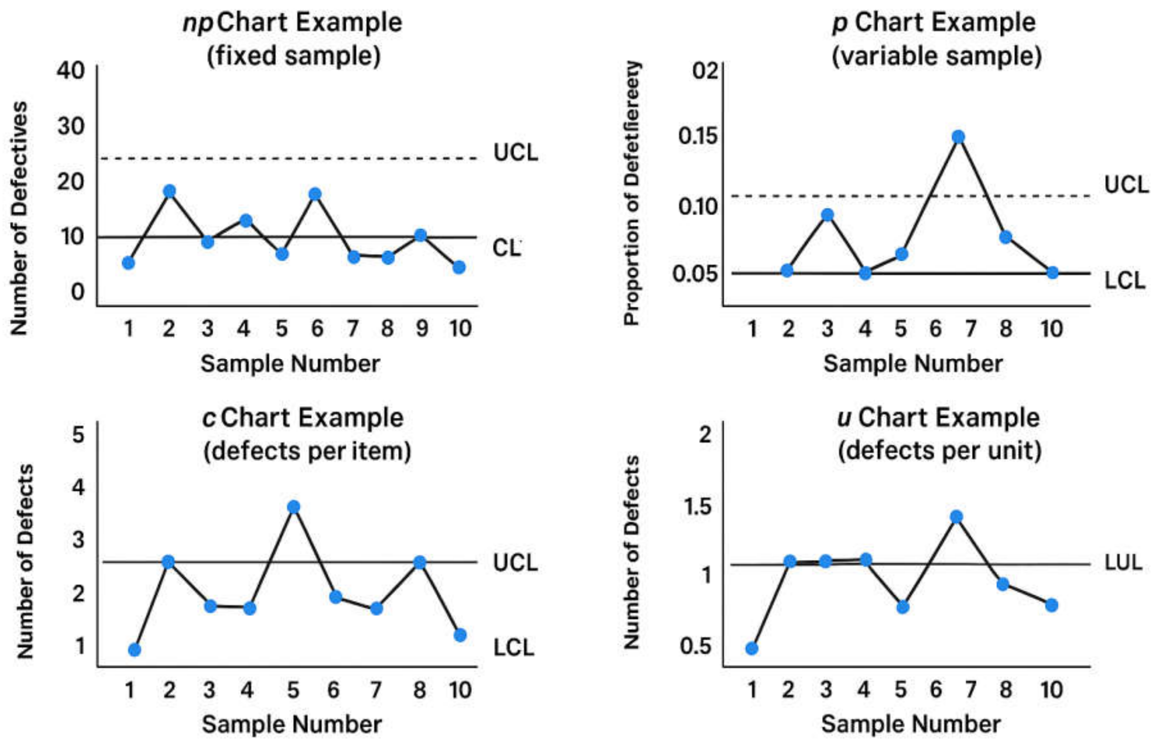
Attribute control charts are used when the data collected from a process are discrete or count-based, meaning the characteristics are judged as either conforming or nonconforming to quality standards. These charts are appropriate when it is impractical or impossible to measure a continuous variable for each unit produced.

Common examples of attribute data include the number of defective products, the proportion of defectives in a batch, or the count of defects per unit. Attribute charts are based on the binomial or Poisson probability distributions, depending on the type of data collected.

The four principal types of attribute control charts are:

1. **np Chart (Number of Defectives Chart):** *(NIST/ITL, n.d.; Perkasa, 2021)*
   - Used when the sample size is constant, and the number of defective items is recorded.
   - Example: Monitoring the number of faulty bolts in each sample of 200 produced.
2. **p Chart (Proportion Defective Chart):** *(Aslam, Khan, & Albassam, 2019; NIST/ITL, n.d.)*
   - Applied when the sample size varies, and the proportion of defectives is measured.
   - Example: Tracking the proportion of defective printed circuit boards in daily samples.
3. **c Chart (Count of Defects Chart):** *(Sałaciński, Chrzanowski, & Chmielewski, 2023)*
   - Suitable when counting the number of defects per inspection unit, assuming a constant area or opportunity for defects.
   - Example: Number of surface scratches per metal sheet inspected.
4. **u Chart (Defects per Unit Chart):** *(Woodall, 2000; NIST/ITL, n.d.)*
   - Used when the inspection unit size varies and the data represent defects per unit.
   - Example: Number of software bugs per 1,000 lines of code across modules of varying lengths.

Attribute charts are easy to construct and interpret, making them suitable for routine quality monitoring. However, their limitation lies in lower sensitivity, as they detect only relatively large shifts in process performance compared to variable charts.



2.2.2 Variable Control Charts *(Montgomery, 2019; Kumar & Joshi, 2023; Chen et al., 2024)*

Variable control charts are used when process data are continuous and measurable on a numerical scale. They provide more detailed information about process behavior, making them ideal for detecting small shifts in mean or variability. These charts assume that the quality characteristic follows an approximately normal distribution.

The two most commonly used variable charts are:

1. **X̄–R Chart (Mean and Range Chart):**
   - The **X̄ Chart** monitors changes in the process mean over time.
   - The **R Chart** tracks the range (difference between maximum and minimum values) within each subgroup, reflecting process dispersion.
   - Widely used for small subgroup sizes ($n \leq 10$) in industries like automotive and electronics.
2. **S Chart (Standard Deviation Chart):**
   - Similar to the R chart but uses the **standard deviation (S)** as a measure of dispersion, making it more accurate for larger subgroup sizes.
   - Suitable for continuous monitoring in high-precision processes such as pharmaceuticals and aerospace manufacturing.

Variable charts are more **statistically sensitive** than attribute charts, allowing early detection of minor shifts that could lead to defects if not corrected. However, they require accurate measurement systems and consistent sampling to ensure reliability.

3. Attribute Control Charts – Concepts and Applications

Attribute control charts play a central role in quality management when the process data are *qualitative or count-based* rather than continuous. In such cases, each product or service outcome is classified as *defective* or *non-defective*, or the *number of defects* is counted per inspection unit. These charts are based on binomial or Poisson probability models, depending on whether the data represent *defectives* (binary outcomes) or *defects* (count data).

A key element in constructing any control chart is the determination of **control limits**, which act as statistical thresholds distinguishing normal process variation from abnormal or assignable causes. Control limits can be established in two ways:

1. **When Standards Are Specified** – limits are based on known or prescribed defect rates defined by industrial standards, customer tolerances, or process capability indices.
2. **When Standards Are Not Specified** – limits are empirically estimated from the observed process data, following Shewhart's principle that control comes from actual process behaviour, not assumed targets.

3.1 np (or) d Chart (Number of Defectives Chart)

The **np (or) d chart** monitors the *number of defective units* in samples of constant size. It is suitable for processes where inspection involves a fixed number of units each time, and the outcome for each unit is either defective or non-defective.

Control Limits

- **Standards Specified:**
  When a predetermined acceptable defect rate $p_0$exists (e.g., regulatory limits in electronics assembly or packaging defect thresholds), the control limits are:

$$CL = np_0,$$

$$UCL = np_0 + 3\sqrt{np_0(1 - p_0)},$$

$$LCL = np_0 - 3\sqrt{np_0(1 - p_0)}$$

These limits indicate the process's expected variation if it meets the target defect rate.

- **Standards Not Specified:**
  When no prior benchmark exists, limits are estimated from sample data:

$$\bar{p} = \frac{\sum \text{defectives}}{\sum n}$$

$$CL = n\bar{p},$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1 - \bar{p})},$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1 - \bar{p})}$$

This method reflects actual process performance rather than theoretical expectations.

Interpretation

If sample counts fall consistently within limits and show random scatter, the process is stable. Any point beyond the limits or systematic pattern (trend, cycle) indicates special cause variation.

Applications

Widely used in electronics, textile, and food packaging industries for monitoring fixed batch outputs—e.g., number of defective circuit boards per 100 tested.

3.2 p Chart (Fraction Defective Chart)

The **p chart** is an extension of the np chart for situations with variable sample sizes. It tracks the *proportion of defective units* rather than their count, allowing consistent evaluation despite changing inspection volumes.

Control Limits

- **Standards Specified:**
  When an acceptable defect fraction $p_0$is known (e.g., ISO tolerance level or Six Sigma target):

$$CL = p_0,$$

$$UCL = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}},$$

$$LCL = p_0 - 3\sqrt{\frac{p_0(1-p_0)}{n}}$$

- **Standards Not Specified:**
  In most industrial applications, control limits are derived from observed process data:

$$\bar{p} = \frac{\sum \text{defectives}}{\sum n}$$

$$CL = \bar{p}$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}},$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

For each sample, limits adjust according to sample size n_i, providing dynamic control boundaries.

Interpretation

A stable p chart indicates that the proportion defective remains consistent with natural variation. Significant deviations imply potential changes in process inputs, supplier quality, or machine calibration.

Applications

Commonly applied in pharmaceutical, consumer goods, and automotive component industries, where batch sizes vary due to production schedules or inspection constraints.

Modern Use

Automated SPC software systems often integrate p charts for real-time batch quality dashboards, enabling process engineers to compare defect rates across shifts and production lines.

3.3 c Chart (Count of Defects per Unit)

The **c chart** tracks the *number of defects* (not defectives) observed within a constant inspection unit, area, or time frame. The data typically follow a Poisson distribution, as defects are random and rare events occurring within fixed opportunities.

Control Limits

- **Standards Specified:**
  If the target defect count per unit is known (c_0), limits are calculated as:

$$CL = c_0,$$

$$UCL = c_0 + 3\sqrt{c_0},$$

$$LCL = c_0 - 3\sqrt{c_0}$$

- **Standards Not Specified:**
  When standards are absent, limits rely on sample data:

$$\bar{c} = \frac{\sum c_i}{k}$$

$$CL = \bar{c},$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}},$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

If $LCL < 0$, it is adjusted to zero since defect counts cannot be negative.

Interpretation

Points exceeding UCL suggest special causes like equipment misalignment or material defects. Sustained trends below CL might indicate over-adjustment or improved quality performance.

Applications

Used extensively in automotive, metal finishing, printing, and aerospace sectors for monitoring surface flaws, paint bubbles, or assembly defects.

Advantages & Modern Extensions

- Easily interpretable in visual inspection environments.
- In advanced systems, AI vision tools now generate c-chart data automatically from camera-based defect detection systems.

3.4 u Chart (Defects per Unit with Variable Sample Size)

The **u chart** generalizes the c chart by allowing variable inspection areas or sample sizes. It plots the *average number of defects per unit*, offering normalized defect density measures.

Control Limits

- **Standards Specified:**
  When the expected defect rate per unit (u_0) is known:

$$CL = u_0,$$

$$UCL_i = u_0 + 3\sqrt{\frac{u_0}{n_i}},$$

$$LCL_i = u_0 - 3\sqrt{\frac{u_0}{n_i}}$$

- **Standards Not Specified:**
  When historical process data are used:

$$\bar{u} = \frac{\sum c_i}{\sum n_i}$$

$$CL = \bar{u},$$

$$UCL_i = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}},$$

$$LCL_i = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$$

Control limits vary for each sample, adapting to changing unit sizes.

Interpretation

When plotted over time, the u chart helps identify periods of abnormal defect densities or inspection irregularities. Variation in sample size can lead to differing control widths, which must be interpreted carefully.

Applications

Ideal for textile, semiconductor, healthcare, and service industries, where inspection opportunities differ (e.g., fabric meters, wafers per batch, or patient cases per day).

Advantages and Industrial Relevance

- Provides standardized monitoring across unequal inspection volumes.
- Commonly used in Statistical Quality Assurance (SQA) for multi-site production analysis, enabling global manufacturers to benchmark defect performance across facilities.

3.5 Comparative Analysis of Attribute Charts and Control Limit Basis

Chart	Data Type	Sample Size	Distribution	When Standards Specified	When Standards Not Specified	Example Applications
np	Defectives	Fixed	Binomial	$UCL = np_0 + 3\sqrt{np_0(1-p_0)}$	$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$	Electronics assembly, packaging
p	Fraction defective	Variable	Binomial	$p_0 \pm 3\sqrt{p_0(1-p_0)/n}$	$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n}$	Food & pharma quality control
c	Defects per item	Fixed	Poisson	$c_0 \pm 3\sqrt{c_0}$	$\bar{c} \pm 3\sqrt{\bar{c}}$	Paint & surface defect monitoring
u	Defects per unit	Variable	Poisson	$u_0 \pm 3\sqrt{u_0/n_i}$	$\bar{u} \pm 3\sqrt{\bar{u}/n_i}$	Textile & semiconductor QC

4. Variable Control Charts – Concepts and Applications

Variable control charts are a core component of Statistical Process Control (SPC), specifically designed to monitor quantitative or measurable data. Unlike attribute charts, which classify items as defective or non-defective, variable charts track continuous measurements such as diameter, weight, temperature, or tensile strength. These charts are particularly useful for detecting small shifts in process mean or variability, enabling timely corrective action before product quality deteriorates.

The central principle behind variable charts is that every process exhibits two dimensions of variation:

1. **Central tendency (mean)** – representing the location or average performance of the process.
2. **Dispersion (range or standard deviation)** – representing the spread or consistency of process data.

By plotting these characteristics over time, variable control charts help quality engineers determine whether the process is stable and predictable (in statistical control) or unstable (affected by assignable causes).

4.1 X̄–R Chart (Mean and Range Chart)

The **X̄–R Chart** is the most widely used variable control chart for subgrouped data with small sample sizes ($n \leq 10$). It consists of two separate but complementary charts:

- **X̄ Chart:** Monitors changes in the process **mean** (central tendency).
- **R Chart:** Monitors changes in the **range** (difference between the maximum and minimum values) within each subgroup, reflecting variability.

Together, these charts provide a comprehensive view of process performance — whether the process average is drifting and whether its variability remains under control.

4.1.1 Purpose and Function

The X̄–R chart is ideal when measurements are taken from subgroups of similar items produced under consistent conditions. It detects small to moderate shifts in both mean and spread and is widely applicable in real-time production monitoring.

Example: In a machining operation producing shafts, five samples are taken every hour. The diameters are measured, the average (X̄) and range (R) are computed, and plotted on the control chart to check whether the machine remains centered and stable.

4.1.2 Control Limits

A. When Standards Are Specified:
If the process standard deviation (σ) or target mean (μ_0) is known from design or historical data, control limits can be set based on those specified parameters.

- For the **X̄ Chart**:

$$CL = \mu_0,$$
$$UCL = \mu_0 + A_2\bar{R},$$
$$LCL = \mu_0 - A_2\bar{R}$$

- For the **R Chart**:

$$CL = \bar{R},$$
$$UCL = D_4\bar{R},$$
$$LCL = D_3\bar{R}$$

Here, A_2, D_3, D_4 are control chart constants that depend on subgroup size (n).

B. When Standards Are Not Specified:
When no preset standard exists, control limits are estimated from sample data:

- Compute the average of subgroup means $\bar{\bar{X}}$ and the average range $\bar{R}$:

$$\bar{\bar{X}} = \frac{\sum \bar{X}_i}{k}, \bar{R} = \frac{\sum R_i}{k}$$

where k is the number of subgroups.

- Then, the control limits are:
    - **For X̄ Chart:**

$$CL = \bar{\bar{X}},$$

$$UCL = \bar{\bar{X}} + A_2\bar{R},$$

$$LCL = \bar{\bar{X}} - A_2\bar{R}$$

    - **For R Chart:**

$$CL = \bar{R},$$

$$UCL = D_4\bar{R},$$

$$LCL = D_3\bar{R}$$

If $LCL_R < 0$, it is set to zero because range cannot be negative.

4.1.3 Interpretation

- **Stable Process:** Points randomly distributed within limits.
- **Mean Shift:** Several points consecutively above or below the center line on the X̄ chart.
- **Increased Variability:** Points beyond UCL on the R chart indicate abnormal spread due to tool wear, machine vibration, or inconsistent materials.

4.1.4 Advantages:

- Provides simultaneous monitoring of mean and variability.
- Simple to construct and widely understood by engineers.
- Effective for detecting both sudden and gradual process changes.

4.1.5 Limitations:

- Less precise for large subgroup sizes (n > 10).
- Assumes normal distribution and constant sample size.

4.1.6 Industrial Applications:

- Machining operations (e.g., shaft diameter, hole depth)
- Assembly lines where subgroup samples are periodically drawn
- Manufacturing tolerance control in automotive, metal, and electronic component production

In modern manufacturing, X̄–R charts are frequently integrated into SPC dashboards, providing live feedback from CNC machines and IoT sensors, thereby enhancing process traceability and predictive maintenance.

4.2 S Chart (Standard Deviation Chart)

The **S Chart** (or **X̄–S Chart**) is used when subgroup sizes are larger than 10. It replaces the range (R) with the standard deviation (S) as a measure of variability. Standard deviation provides a more reliable and statistically efficient estimate of process dispersion, especially when subgroup sizes are large.

Like the X̄–R chart, the X̄–S chart consists of two parts:

- **X̄ Chart:** Monitors process mean.
- **S Chart:** Monitors process variability (based on standard deviation rather than range).

4.2.1 Control Limits

A. When Standards Are Specified:
If target mean (μ_0) and standard deviation (σ_0) are known:

- **For X̄ Chart:**

$$CL = \mu_0, UCL = \mu_0 + A_3\bar{S}, LCL = \mu_0 - A_3\bar{S}$$

- **For S Chart:**

$$CL = \bar{S}_0, UCL = B_4\bar{S}_0, LCL = B_3\bar{S}_0$$

B. When Standards Are Not Specified:
When calculated from sample data:

1. Compute the overall mean of subgroup means $\bar{\bar{X}}$and the average standard deviation $\bar{S}$:

$$\bar{\bar{X}} = \frac{\sum\bar{X}_i}{k}, \bar{S} = \frac{\sum S_i}{k}$$

2. Determine the control limits:
   - **For X̄ Chart:**

$$UCL = \bar{\bar{X}} + A_3\bar{S}, CL = \bar{\bar{X}}, LCL = \bar{\bar{X}} - A_3\bar{S}$$

   - **For S Chart:**

$$UCL = B_4\bar{S}, CL = \bar{S}, LCL = B_3\bar{S}$$

The constants $A_3, B_3,$and B_4depend on subgroup size (n) and are available in SPC reference tables.

4.2.2 Interpretation

- The S chart provides smoother and more accurate estimation of variability than the R chart, particularly for large subgroups.
- Consistent S values indicate stable process dispersion; increasing or fluctuating values suggest process inconsistency.
- The $\bar{X}$ chart identifies shifts in process mean that could result from tool wear, calibration drift, or raw material inconsistency.

4.2.3 Advantages:

- More accurate than R charts for large n (>10).
- Better statistical representation of dispersion.
- Suitable for processes requiring tight tolerance and precision.

4.2.4 Limitations:

- More complex calculations compared to range-based methods.
- Requires computational tools or SPC software for real-time application.

4.2.5 Industrial Applications:

- **Precision engineering:** Tolerances in aerospace or optical component manufacturing.
- **Pharmaceutical production:** Monitoring concentration, viscosity, or tablet weight consistency.
- **Chemical processes:** Tracking variation in pH, temperature, or reaction time.

With the rise of digital manufacturing systems, S charts are now embedded in automated quality monitoring software, allowing data from multiple sensors to be aggregated and analyzed in real time.

4.3 Comparative Table of Variable Charts

Chart Type	Subgroup Size (n)	Monitored Parameter	Data Type	Common Use / Industry
$\bar{X}$–R Chart	≤ 10	Process Mean & Range	Continuous	General process monitoring, machining, assembly lines
$\bar{X}$–S Chart	> 10	Process Mean & Standard Deviation	Continuous	High-precision industries such as pharmaceuticals, aerospace, and electronics

5. Comparative Overview of Attribute and Variable Charts *(Bersimis & Psarakis, 2023; Castagliola et al., 2024)*

Aspect	Attribute Control Charts	Variable Control Charts
Data Type	Discrete or count-based data (e.g., number of defectives, proportion	Continuous or measurable data (e.g., diameter, weight, temperature, tensile strength).

	defective, or number of defects per unit).	
Typical Statistical Distribution	Binomial or Poisson distributions depending on whether the chart tracks defectives (p, np) or defects (c, u).	Assumed to follow a normal distribution for sample means and variability measures ($\bar{X}$–R, $\bar{X}$–S).
Monitored Characteristic	Quality classification or frequency of non-conformance.	Actual measurement of process characteristic (mean and variation).
Sensitivity to Process Changes	Lower - detects major process shifts or trends in defect counts.	Higher - detects minor variations in mean or standard deviation, providing early warning signals.
Type of Data Collection	Based on inspection outcomes or defect counts (go/no-go data).	Based on quantitative measurements obtained through instruments or sensors.
Sample Size Requirement	Larger sample sizes needed for reliable estimation due to discrete nature of data.	Smaller sample sizes sufficient because continuous data provide more information per observation.
Control Limit Basis	Derived from binomial or Poisson probabilities (e.g., $\pm 3\sigma$ limits around defect proportion).	Derived from process mean and standard deviation using constants (A_2, A_3, D_3, D_4, B_3, B_4).
Ease of Construction	Simple to construct and interpret; minimal computation required.	More complex; requires statistical calculation of subgroup means, ranges, or standard deviations.
Cost of Implementation	Low, as only counts or classifications are required.	Relatively higher, requiring measurement tools and trained personnel.
Sensitivity to Measurement Error	Low sensitivity since classification errors are infrequent.	High sensitivity; measurement precision directly affects chart accuracy.
Industrial Applicability	Preferred in service operations, assembly lines, textiles, packaging, and consumer goods manufacturing.	Common in machining, aerospace, automotive, electronics, chemical, and pharmaceutical processes.
Examples of Charts	p, np, c, and u charts.	$\bar{X}$–R and $\bar{X}$–S charts.

Typical Interpretation	Indicates the proportion or number of nonconformities in a sample.	Indicates variation in central tendency and spread of measured data.
Best Suited For	Processes where measurement is difficult or cost-prohibitive, or when quality is determined by inspection outcomes.	Processes where precision measurements are available and quality is defined by numerical tolerances.

6. Challenges and Research Gaps

Although Statistical Process Control (SPC) has been successfully applied for decades, several limitations persist in adapting classical methods to modern, data-driven manufacturing environments.

6.1 Handling of Autocorrelated and Non-Normal Data

Traditional SPC charts ($\bar{X}$–R, p, c, u) assume that observations are independent and normally distributed. However, process data in continuous or automated systems often show autocorrelation and non-normality due to sensor feedback, automated control loops, or physical process constraints.

- Such conditions lead to unreliable control limits and false alarms.
- Though corrective methods like residual charting and data transformation exist, they are not fully standardized or widely implemented.
  Hence, there is a need for robust SPC frameworks that can manage correlated, skewed, or complex data patterns.

6.2 Real-Time SPC Integration with IoT and AI

Conventional SPC was designed for offline analysis, while modern industries require real-time monitoring supported by IoT sensors and AI algorithms.

- Current systems struggle with large data volumes, integration issues, and latency in decision-making.
- Few industrial implementations achieve full automation from data collection to corrective action.
  Developing AI-enabled, real-time SPC systems capable of adaptive control and predictive alerts remains an open research challenge.

6.3 Limited Studies on Mixed Attribute–Variable Chart Integration

Manufacturing environments frequently involve both measurement-based (variable) and count-based (attribute) quality characteristics. Despite this, most studies treat these chart types separately.

- Research on hybrid SPC systems capable of analyzing both defect counts and measurement variability is scarce.
- Comparative models integrating these chart types for holistic quality assessment are needed, especially in hybrid industries such as automotive and electronics.

6.4 Need for Adaptive and Self-Learning Control Limits

In smart factories, process parameters change dynamically due to automation, material variability, or machine learning-based optimization. Static $\pm 3\sigma$ limits are insufficient for such adaptive environments.

- Self-learning control charts that adjust control limits based on historical and real-time data can improve responsiveness.
- However, their interpretability, computational efficiency, and industrial validation remain limited.
  Research should focus on adaptive SPC systems that balance automation with operator understanding and trust.

7. Emerging Trends and Future Directions

7.1 Multivariate Control Charts

Processes often involve interrelated variables where univariate charts are inadequate. Multivariate SPC methods such as Hotelling's T^2, MEWMA, and PCA-based SPC allow simultaneous monitoring of correlated quality characteristics. These methods are particularly useful in chemical, semiconductor, and automotive industries where multidimensional control is critical.

7.2 Machine Learning–Enhanced SPC

Combining SPC with machine learning (ML) techniques enables predictive quality management. ML algorithms (e.g., SVM, Random Forests, Neural Networks) can detect nonlinear relationships and predict process shifts before they occur.

- Integrating ML with SPC transforms monitoring from reactive to predictive, enhancing early warning capabilities.
- Such systems can automatically identify root causes of variation, improving preventive maintenance and process optimization.

7.3 Adaptive and Bayesian SPC Models *(Bersimis et al., 2023; Garg et al., 2025; Zhou et al., 2024)*

Adaptive SPC dynamically updates control limits based on new data, improving sensitivity in changing environments.
Bayesian SPC incorporates prior process knowledge, allowing probabilistic updates and better handling of uncertainty.
Both approaches support continuous learning, aligning with the goals of Industry 4.0 where systems evolve with process conditions.

7.4 Integration with Big Data, IIoT, and Cyber-Physical Systems *(MDPI, 2023; Machines, 2024)*

The convergence of SPC with Big Data analytics and Industrial Internet of Things (IIoT) enables continuous, high-volume process monitoring.

- **Cyber-Physical Systems (CPS)** link real-world processes with digital simulations, supporting real-time quality control through digital twins.
- **Big Data–driven SPC** enhances decision accuracy by analyzing vast, heterogeneous data streams from sensors and machines.

Future SPC systems will be cloud-integrated, intelligent, and self-correcting, turning data into actionable insights for process improvement.

8. Conclusion

Statistical Process Control continues to be a cornerstone of quality management, helping organizations ensure process stability and defect-free production. This review emphasizes that attribute and variable control charts play complementary roles in modern manufacturing systems.

- **Attribute charts** (p, np, c, u) efficiently track defect counts or proportions, making them practical for high-volume inspection processes.
- **Variable charts** ($\bar{X}$–R, S) offer greater precision and sensitivity, ideal for continuous process monitoring and small-shift detection.

Together, they provide a balanced framework for both inspection-based conformance and process-based improvement.

However, the transition to smart manufacturing and Industry 4.0 demands an evolution from traditional SPC to intelligent, adaptive, and data-driven control systems. Integrating SPC with AI, IoT, and real-time analytics can transform quality management from a reactive to a predictive and prescriptive discipline.

In the future, SPC systems will not merely monitor processes but will learn, predict, and optimize them autonomously — creating self-regulating, self-improving quality ecosystems that define the next era of manufacturing excellence.

References

1. Malindžáková, M., Čulková, K., & Trpčevská, J. (2023). *Shewhart control charts implementation for quality and production management.* Processes, 11(4), 1246. https://doi.org/10.3390/pr11041246 mdpi.com
2. “Interpreting statistical process control (SPC) charts using machine learning and expert system techniques.” IEEE Conference Publication. IEEE Xplore. IEEE Xplore
3. Perkasa, L. (2021). *Benefit using statistical process control (SPC) for process control in textile manufacturing: A review.* Journal of Mechanical Science and Engineering, 8(1), 23-28. https://doi.org/10.36706/jmse.v8i1.54 jmse.ejournal.unsri.ac.id+1
4. “Empowering manufacturing environments with process mining-based statistical process control.” Machines, 12(6), 411. MDPI. mdpi.com
5. Sałaciński, T., Chrzanowski, J., & Chmielewski, T. (2023). *Statistical process control using control charts with variable parameters.* Processes, 11(9), 2744. https://doi.org/10.3390/pr11092744 mdpi.com
6. “Statistical quality and process control in biopharmaceutical manufacturing – practical issues and remedies.” (2021). PubMed. PubMed
7. NIST/ITL (n.d.). *What are attributes control charts?* In NIST/ITL Handbook. Retrieved from https://www.itl.nist.gov/div898/handbook/pmc/section3/pmc33.htm