# K Power 3 Heronian Mean Labeling of Special Graphs

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**Abstract:**

This article introduces an innovative concept in graph labeling called K-Power 3 Mean Heronian Mean Labeling. This method merges characteristics of K - Power labeling with the Heronian Mean to create a new way of assigning labels to graph nodes. It establishes a connection between the labels of neighboring nodes, where the label of each vertex is determined by a function that incorporates the powers of its neighbor's label and their Heronian Mean. We delve into the theoretical underpinnings of this labeling technique, explore its mathematical properties, and present findings for various classes of graphs. Furthermore, we discuss the potential applications of K-Power 3 Heronian Mean Labeling in fields such as network design, error correction, and other areas within combinatorics and graph theory. Our results underscore the adaptability and intricacy of this new labeling method, providing valuable insights into the structure and dynamics of labeled graphs.

**Keywords:**

Power 3 Heronian Mean labeling, Silhouette graph, line graph of the complement, Split graph, Polygonal chain.

## 1.Introduction :

Graph labeling is a well-established area in combinatorics, where integers are assigned to vertices or edges of a graph following specific rules or constraints. Labeling schemes have found applications in diverse areas such as coding theory, network design, and circuit layout optimization. Traditional graph labeling techniques, such as edge labeling or vertex labeling, often aim to assign labels to ensure certain mathematical properties are satisfied, such as minimizing conflicts or optimizing network performance. One class of graph labeling that has received attention is K-Power Labeling, which involves assigning labels to the vertices such that the label of each vertex is related to the power of the labels of its neighbors. In this study, we extend this concept by introducing the K-Power 3 Mean Heronian Mean Labeling where the label of a vertex depends on a combination of the labels of its neighbors raised to the third power and the Heronian Mean.

**Definition: 1.1**

A Power 3 Heronian Graph is a special class of graph with specific labeling conditions that involve assigning distinct integers from 1 to q+1 (where q is the number of edges) to the vertices of the graph, edge label that is defined as a function '$\varphi'$ as $\varphi\ (\alpha)$, for vertex α and $\varphi\ (\beta)$ for vertex β across

$$\varphi(e=\alpha\beta)=\left\lceil\left(\frac{\varphi(\alpha)^3+(\varphi(\alpha)\varphi(\beta))^{\frac{3}{2}}+\varphi(\beta)^3}{3}\right)^{\frac{1}{3}}\right\rceil \ or\ \left\lfloor\left(\frac{\varphi(\alpha)^3+(\varphi(\alpha)\varphi(\beta))^{\frac{3}{2}}+\varphi(\beta)^3}{3}\right)^{\frac{1}{3}}\right\rfloor$$

These labeling conditions generally focus on the concept of edge weights determined by the Heronian mean and are usually applied in contexts related to graph invariants. It is a labeling scheme that assigns values of vertices and edges using the Power 3 Heronian mean characteristic-functional equivalent labels.

**Definition: 1.2**

A $k$ – Power 3 Heronian Mean Labeling of a graph with $p$ vertices and $q$ edges $\varphi$ is a function from the node set to $\{k, k+1, \ldots\ldots, k+q\}$ be an injective function and the induced edge labelling be defined by

$$\varphi = \left\lceil \left( \frac{\varphi(\alpha)^3 + (\varphi(\alpha)\varphi(\beta))^{\frac{3}{2}} + \varphi(\beta)^3}{3} \right)^{\frac{1}{3}} \right\rceil \ or \ \left\lfloor \left( \frac{\varphi(\alpha)^3 + (\varphi(\alpha)\varphi(\beta))^{\frac{3}{2}} + \varphi(\beta)^3}{3} \right)^{\frac{1}{3}} \right\rfloor$$

with distinct edge labels.

## 2. Main Outcomes :

**Theorem: 2.1:**

Shadow or Silhouette graph of Linear $S(P_n)$ is a K Power 3 Heronian Mean graph.

**Proof:**

In certain labeling problems, a shadow graph might refer to a graph where the labeling constraints or relationships have been relaxed or modified. It could represent a simplified version of the original graph, emphasizing specific features like node degrees or it might display a subset of the original graph.

Let $u_i$ ; $i\ varies\ drom\ 1\ to\ n$ be the nodes of a Linear $P_n$ and $v_i$ ; $i\ varies\ from\ 1\ to\ n$ be the node corresponding to the nodes $u_i$ ; $i\ varies\ from\ 1\ to\ n$ in order to obtain $S(P_n)$. The graph have $2n$ nodes and $4(n - 1)$ edges.

The function $\varphi$ is defined as a mapping from the node set to the set $\{k, k + 1, k + 2, \ldots k + q\}$, where $\varphi$ assigns each node in $S(P_n)$ an element from the specified range of integers, starting at $k$ and ending at $k + q$.

$$\varphi(\alpha_1) = k \ ; \ \varphi(\alpha_2) = k + 1$$

$$\varphi(\alpha_i) = k + 3(i - 1), \ i\ varies\ from\ 3\ to\ n - 1$$

$$\varphi(\alpha_n) = k + 3(i - 3)$$

$$\varphi(\beta_1) = k + 2 \ ; \ f(\beta_i) = k + 2(i + 2), \ i\ varies\ from\ 2\ to\ n.$$

Induced edges are assigned with

$$\varphi(\alpha_i\alpha_{i+1}) = k + 2i - 1, \ i\ varies\ from\ 2\ to\ n - 1$$

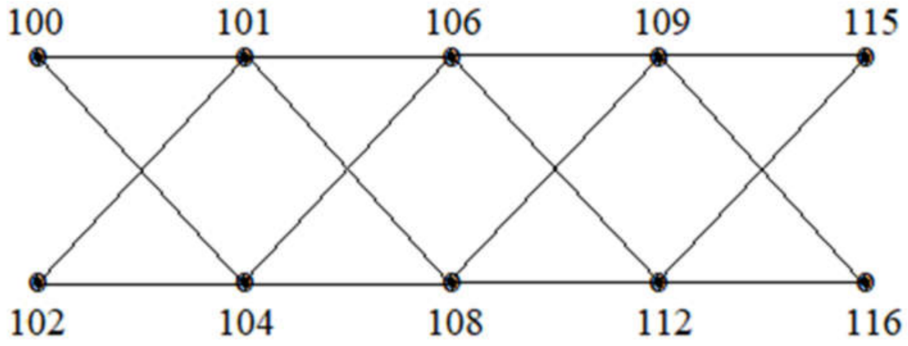
$$\varphi(\beta_i\beta_{i+1}) = k + 2i - 3, \ i\ varies\ from\ 1\ to\ n - 1$$

$$\varphi(\alpha_i\beta_{i+1}) = k + 2i - 4, \quad i \ varies \ from \ 1 \ to \ n$$

$$\varphi(\alpha_{i+1}\beta_i) = k + 2i - 2, \quad i \ varies \ from \ 1 \ to \ n - 1$$

The assigned node and edge labels are distinct. Hence $S(P_n)$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 2.2:** 100 - Power 3 Heronian Mean $S(P_5)$




**Theorem 2.3:**

Total graph or line graph of the complement of linear $T(P_n)$ is a K Power 3 Heronian mean graph.

**Proof:**

A total graph or line graph of the complement is a graph that includes both the nodes and the arcs of $G$ as part of its structure. $T(P_n)$ consist of all the nodes and arcs of the original graph $G$. The arcs of $T(P_n)$ connect two nodes from $G$, if those nodes are adjacent in $G$. A node and an edge from $G$, if the node is incident to that edge in $G$.

Let $u_1u_2 \dots \dots u_n$ be the nodes and $e_1e_2 \dots \dots e_{n-1}$ be the arcs of linear $P_n$ with nodes $2n - 1$ and arcs $4n - 5$.

The function $\varphi$ is defined as a mapping from the node set to the set $\{k, k + 1, k + 2, \dots k + q\}$, where $\varphi$ assigns each node in $T(P_n)$ an element from the specified range of integers, starting at $k$ and ending at $k + q$.

$$\varphi(\alpha_i) = k + 4(i-1), \ i \ varies\ from\ 1\ to\ n-1$$

$$\varphi(\beta_i) = k + 2(2i-3), \ i \ varies\ from\ 1\ to\ n.$$

Induced edges are labeled with

$$\varphi(\alpha_i\alpha_{i+1}) = k + 3i, \ i \ varies\ from\ 1\ to\ n-1$$

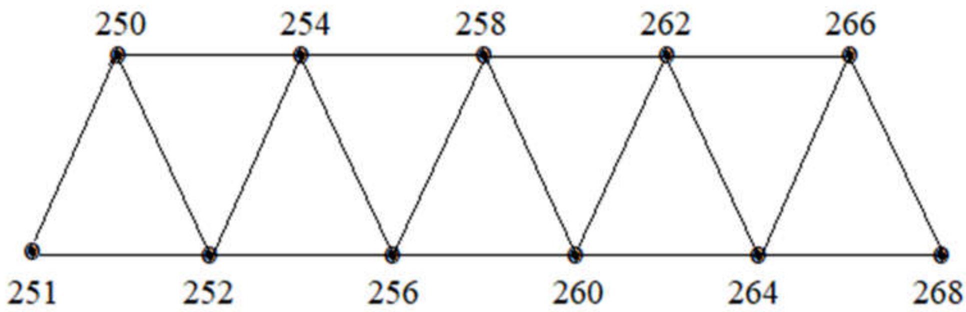
$$\varphi(\beta_i\beta_{i+1}) = k + 2(2i-3), \ i \ varies\ from\ 1\ to\ n-1$$

$$\varphi(\alpha_i\beta_{i+1}) = k + 2(2i-4), \ i \ varies\ from\ 1\ to\ n$$

$$\varphi(\alpha_{i+1}\beta_i) = k + 2(2i-2), \ i \ varies\ from\ 1\ to\ n-1$$

The assigned node and edge labels are distinct. Hence $T(P_n)$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 2.4:** 250 Power 3 Heronian mean graph labeling $T(P_6)$




**Theorem: 2.5**

Splitting or split graph of Path $Sp(P_n)$ is a K Power 3 Heronian mean graph.

**Proof :**

A splitting graph refers to the process of transforming or splitting an existing graph into a new one by changing its structure. There are various specific types of splitting graphs that depend on the context, but a common approach involves splitting nodes or edges based on certain conditions.

Let $u_1 u_2 \ldots \ldots u_n$ be the nodes and $e_1 e_2 \ldots \ldots e_{n-1}$ be the arcs of linear graph . Let $v_1 v_2 v_3 \ldots \ldots v_n$ be the nodes to form the Splitting or split graph of linear graph with nodes $2n$ and arcs $3n - 3$.

The function $\varphi$ is defined as a mapping from the node set to the set $\{k, k + 1, k + 2, \ldots k + q\}$, where $\varphi$ assigns each node in $Sp(P_n)$ an element from the specified range of integers, starting at $k$ and ending at $k + q$.

$$\varphi(\alpha_1) = k \ ; \quad \varphi(\alpha_2) = k + 1$$

$$\varphi(\alpha_i) = k + 3(i - 1)\,, \ i \ varies\ from\ 3 \ \ to\ n - 1$$

$$\varphi(\beta_i) = k + 2i + 1 \ ; \ , \quad i \ varies\ from\ 1\ to\ 2$$

$$\varphi(\beta_i) = k + 3i - 1 \ , \ i \ varies\ from\ 3\ to\ n - 1.$$

Induced arcs are assigned with

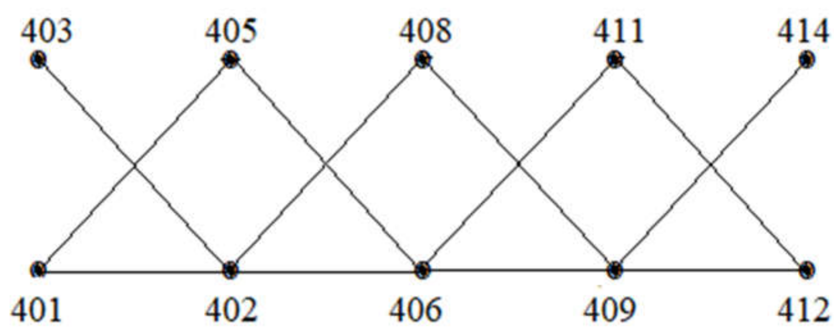
$$\varphi(\alpha_i \beta_{i+1}) = k + 2(2i - 1), \ i \ varies\ from \ 1 \ to\ n - 1$$

$$\varphi(\alpha_{i+1} \beta_i) = k + 2(2i - 2), \ i \ varies\ from \ 1\ to\ n - 1$$

$$\varphi(\beta_{i+1} \beta_i) = k + 2(2i - 3), \ i \ varies\ from \ 1\ to\ n - 1$$

The assigned node and edge labels are distinct. Hence $Sp(P_n)$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 2.6:** 401 Power 3 Heronian mean labeling $Sp(P_6)$




**Theorem: 2.7**

Polyline $G_{m,n}$ are Power 3 mean graph for all nodes and segments.

**Proof :**

A cycle polygonal chain is a specific type of polygonal chain where the chain is closed, meaning the first and last vertices are connected, forming a closed loop. It is essentially a polygonal chain that forms a cycle.

A cycle polygonal chain is a polygonal chain where the first and last vertices are connected by a line segment, forming a closed path. It consists of a sequence of vertices and edges where each edge connects consecutive vertices, and the last vertex is connected back to the first vertex, completing the cycle.

In $G_{m,n}$, m could represent the number of points or vertices, and nnn would denote the number of line segments (edges) connecting these points.

The function $\varphi$ is defined as a mapping from the node set to the set $\{k, k + 1, k + 2, \ldots k + q\}$, where $\varphi$ assigns each node in $G_{m,n}$ an element from the specified range of integers, starting at $k$ and ending at $k + q$.

$$\varphi(\alpha_i) = k + i\ ,\ i\ varies\ from\ 3\ \ to\ mn - 1$$

$$\varphi(\alpha_n) = mn + 1$$

Then the labels of the edges are given below,

$\varphi(\alpha_{mn+1}\alpha_{mn+2}) = k + mn + 1$

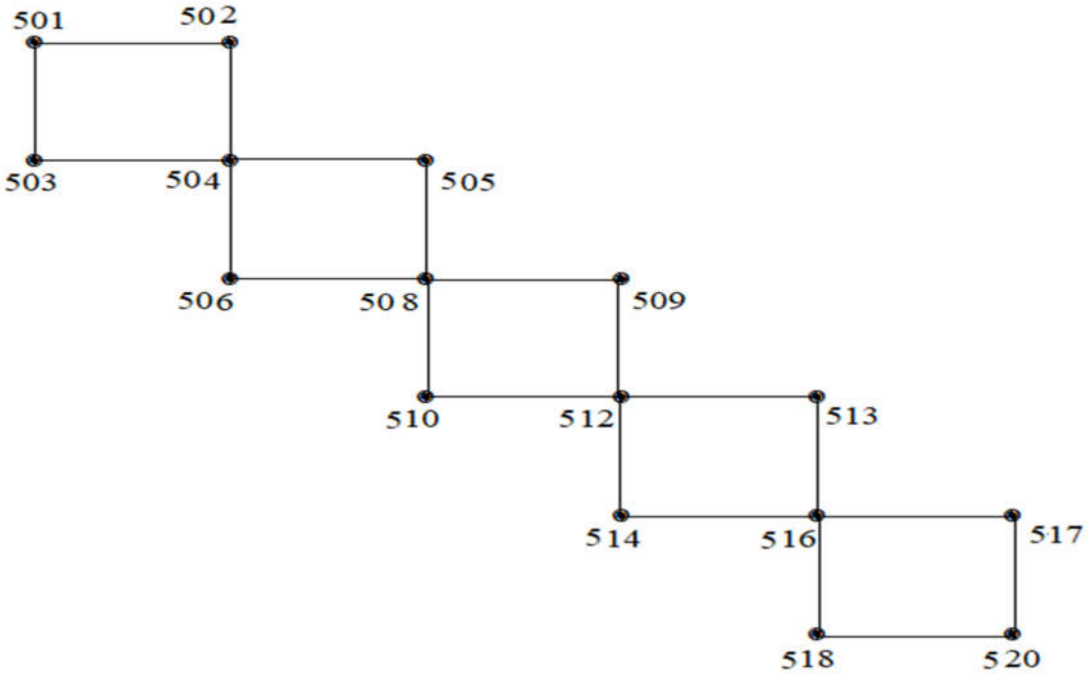
$\varphi(\alpha_{mn+i}\alpha_{mn+i+2}) = k + mn + i + 1$

$\varphi\big(\alpha_{(m+1)n-2}\alpha_{(m+1)n+1}\big) = k + (m + 1)n - 1$

$\varphi\big(\alpha_{(m+1)n+1}\alpha_{(m+1)n-1}\big) = k + (m + 1)n.$

The assigned node and edge labels are distinct. Hence $G_{m,n}$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 2.8:** 501 Power 3 Heronian mean labeling Polyline $G_{m,n}$




**Theorem: 2.9**

$D(T_n) \odot K_1$ is a $k$ -Power 3 Heronian mean graph for each n.

**Proof:**

Double Triangular Snake is denoted by $D(T_n)$. $a_i, b_i, c_i$ be the nodes of a Double Triangular Snake. connect $a_i b_i, a_{i+1} b_i, a_i c_i$ and $a_{i+1} b_i$ . Let $p_i, q_i$ and $r_i, s_i$ be the vertices that are pendant connect $a_i p_i, b_i q_i$ and $a_i r_i, c_i q_i$.

$$\varphi : V\ (D(T_n) \odot K_1) \rightarrow \{k, k+1, k+2, \ldots\ldots, k+q\} \text{ by}$$

$$\varphi(a_i) = k + 3(3i+1), i\ varies\ from\ 1\ to\ \ n$$

$$\varphi(b_i) = k + 3(3i+2), i\ varies\ from\ 1\ to\ \ n-1,$$

$$\varphi(p_i) = k + 3(3i+4), i\ varies\ from\ 1\ to\ \ n-1,$$

$$\varphi(q_i) = k + 3(3i+3), \quad i\ varies\ from\ 1\ to\ \ n,$$

$$\varphi(r_i) = k + 3(3i+8), \quad i\ varies\ from\ 1\ to\ \ n-1,$$

$$\varphi(s_i) = k + 3(3i + 10), \quad i \text{ varies from } 1 \text{ to } \ n$$

$$\varphi(s_i) = k + 3(i + 8), i \text{ varies from } 1 \text{ to } \ n - 1,$$

$$\varphi(a_i b_{i+1}) = k + 3(3i + 4), \ i \text{ varies from } 1 \text{ to } \ n - 1,$$

$$\varphi(a_i b_i) = k + 3(3i + 10), i \text{ varies from } 1 \text{ to } \ n - 1,$$

$$\varphi(a_{i+1} b_i) = k + 3(3i + 2), i \text{ varies from } 1 \text{ to } \ n - 1,$$

$$\varphi(a_i c_i) = k + 3(3i + 8), i \text{ varies from } 1 \text{ to } \ n - 1$$

$$\varphi(a_{i+1} c_i) = k + 3(9i + 1), \ i \text{ varies from } 1 \text{ to } \ n - 1,$$

$$\varphi(a_i p_i) = k + 3(3i + 10), i \text{ varies from } 1 \text{ to } \ n$$

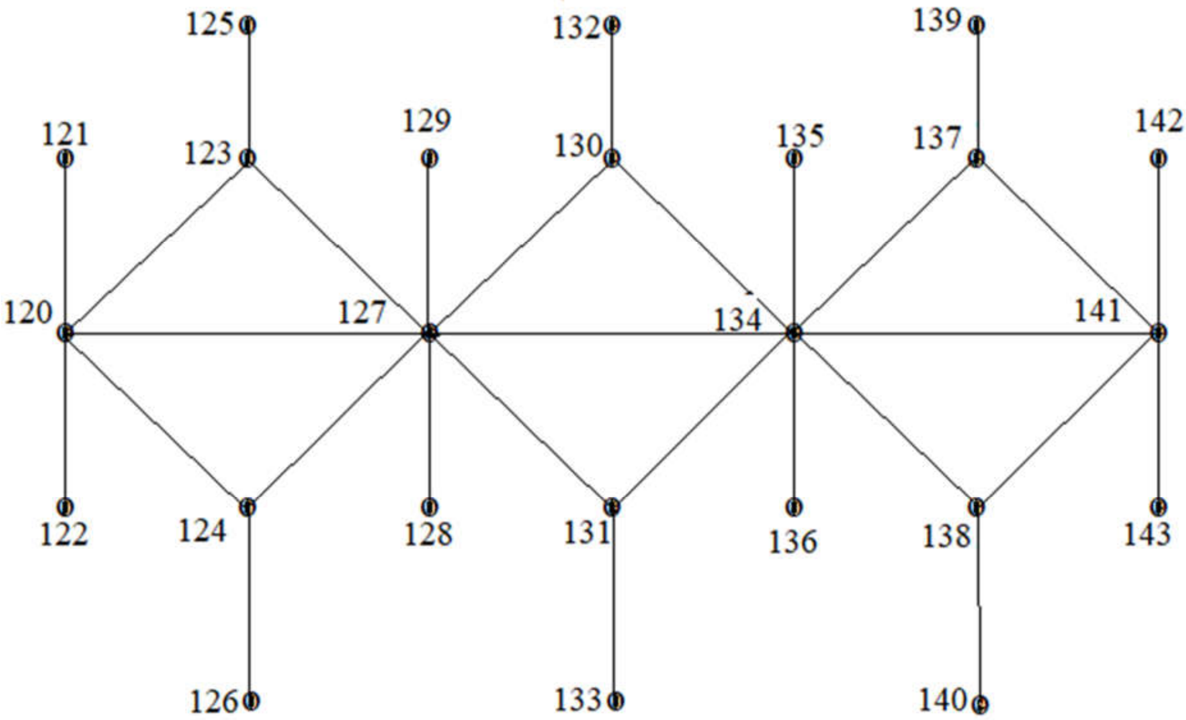
$$\varphi(a_i q_i) = k + 3(i + 14), i \text{ varies from } 1 \text{ to } \ n - 1,$$

$$\varphi(b_i s_i) = k + 3(3i + 10), i \text{ varies from } 1 \text{ to } \ n$$

$$\varphi(c_i r_i) = k + 3(3i + 6), i \text{ varies from } 1 \text{ to } \ n - 1.$$

The assigned node and edge labels are distinct. Hence $D(T_n) \odot K_1$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 2.10:** 120 -Power 3 Heronian labelling of $D(T_4) \odot K_1$




**Theorem:2.11**

Any crown $C_n \odot K_1$ a $k$ -Power 3 Heronian mean graph for each $n \geq 3$.

**Proof:**

A crown is a specific type of graph, often used in the context of certain types of graph operations or graph families. A graph is typically a graph that consists of two parts A cycle or circular arrangement of vertices. A set of pendant edges (or leaves) attached to vertices of the cycle. More specifically, a crown graph is a graph that consists of a cycle $C_n$, which is a simple polygon formed by nnn vertices connected in a loop. Each vertex in the cycle is attached to one or more pendant vertices (vertices of degree 1), which are connected by edges to the cycle.

Let $C_n$ be a cycle $u_1 u_2 \ldots \ldots u_n u_1$.Let $v_i$ be the pendent vertices adjacent to $u_i$ ; $i$ $varies\ from\ from$ 1 $to\ n$.

Define a function $\varphi : V(C_n \odot K_1) \rightarrow \{k, k + 1, k + 2, \ldots k + q\}$ by

$$\varphi(\alpha_i) = k + 2i - 1, \ i \ varies \ from \ 1 \ to \ n$$

$$\varphi(\beta_i) = k + 2i, \ i \ varies \ from \ 1 \ to \ n$$

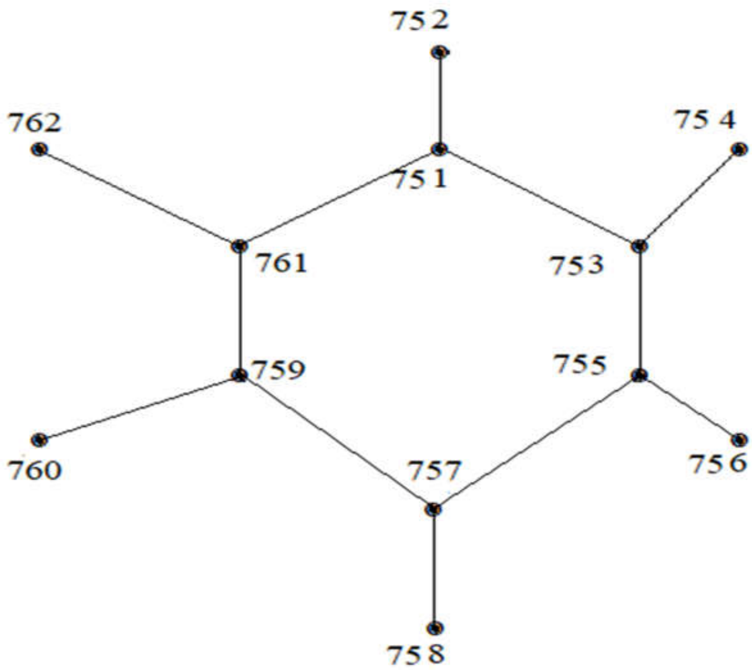
Induced arcs are assigned with

$$\varphi(\alpha_i \alpha_{i+1}) = k + 2(2i - 1), \ i \ varies \ from \ 1 \ to \ n$$

$$\varphi(\alpha_i \beta_i) = k + 2(2i - 2), \ i \ varies \ from \ 1 \ to \ n$$

The assigned node and edge labels are distinct. Hence $D(T_n) \odot K_1$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 2.12:** 751 - Power 3 Heronian labelling of $C_n \odot K_1$



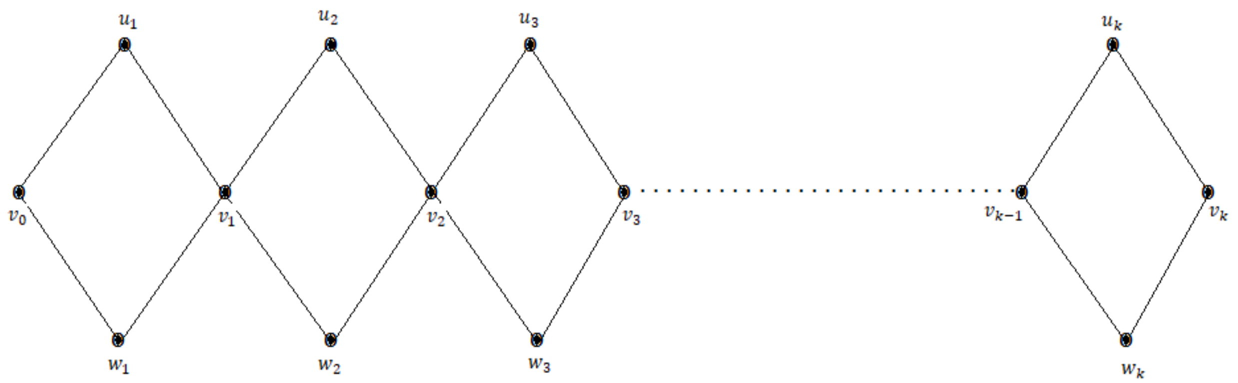

**3. K Power 3 Heronian Mean Labeling of Snake Graphs**

**Definition: 3.2**

Diamond snake graph is a specific type of graph that can be constructed using a combination of the diamond and snake structures. While the exact definition of the "diamond snake graph" may vary slightly depending on the context, it generally refers to a graph that has a repetitive structure resembling a snake with some additional diamond shapes or motifs within it.

A diamond snake graph consists of several diamond graphs (connected cycles) connected in a linear (snake-like) fashion. It can be imagined as a snake where each segment of the snake is a diamond. More formally, a diamond snake graph can be defined as a sequence of diamond graphs (each having 4 vertices and 5 edges). These diamond graphs are connected in such a way that the resulting structure forms a path or a chain-like structure, giving the graph its snake appearance.

A Diamond Snake graph is constructed by starting with a path $v_0 v_1 \ldots . v_k$ , and for each pair of consecutive vertices $v_i$ and $v_{i+1}$ (where i varies from 0 to $k-1$ ), two new vertices $u_{i+1}$ and $w_{i+1}$ are added and connected to $v_i$ and $v_{i+1}$, forming a cycle $C_4$ for each edge of the path. Thus, each edge in the original path is replaced by a 4-cycle, and the distance between nodes $v_i$ and $v_{i+1}$is 2.




**Theorem : 3.3**

Diamond Snake Graph $kC_4$ admits a $k$ -Power 3 Heronian mean graph.

**Proof:**

Let $G = kC_4$ is a graph obtained from a path $v_0 v_1 v_2 \ldots \ldots v_k$ by joining vertices $v_i$ and $v_{i+1}$ to two new vertices $u_{i+1}$ and $w_{i+1}$ for $0 \le i \le k-1$. That is every edge of a path $v_0 v_1 v_2 \ldots \ldots v_k$ of size $k$ is replaced by a cycle $C_4$ and $d(v_i v_{i+1}) = 2$.

A Diamond Snake graph has $3k+1$ vertices and $4k$ edges where $k$ is the number of blocks in $kC_4$. Let $V = \{v_0 w_1 v_1 w_2 v_2 \ldots \ldots w_k v_k\}$ be the set of vertices of $kC_4$.

Define a function $\varphi : V(kC_4) \to \{k, k+1, k+2, \ldots k+q\}$ by

$$\varphi(\alpha_i) = k + 4i - 3 \, ; \; i \; varies \; from \; 1 \; to \; n$$

$$\varphi(\beta_1) = 2; \; \varphi(\beta_i) = k + 4i \, ; \; i \; varies \; from \; 2 \; to \; n$$

$$\varphi(\gamma_i) = k + 4i - 1 \, ; \; i \; varies \; from \; 1 \; to \; n$$

Induced edges are labeled with

$$\varphi(\alpha_i \beta_i) = k + 4i - 3 \, , \quad i \; varies \; from \; 1 \; to \; n$$

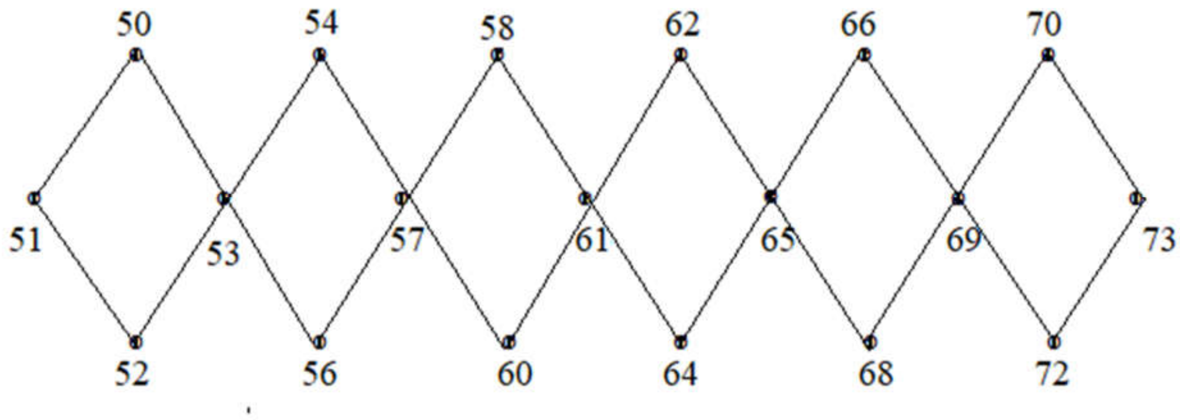
$$\varphi(\alpha_i \beta_{i+1}) = k + 4i - 1, \quad i \; varies \; from \; 1 \; to \; n$$

$$\varphi(\beta_i \gamma_i) = \mathrm{k} + 4\mathrm{i} - 2 \, ; \quad i \; varies \; from \; 1 \; to \; n$$

$$\varphi(\beta_{i+1} \gamma_i) = \mathrm{k} + 4\mathrm{i} \, ; \quad i \; varies \; from \; 1 \; to \; n$$

The above defined function $\varphi$ provides $k$-Power 3 Heronian Mean labeling of the graph. Hence $kC_4$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 3.4 :** 50 - Power 3 Heronian labelling of $6C_4$

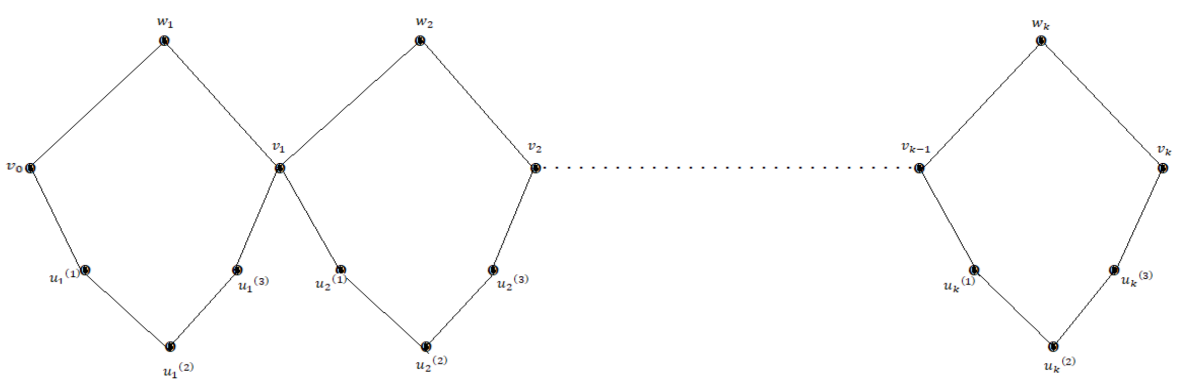



**Definition: 3.5**

A hexagonal snake refers to a graph that consists of multiple hexagonal (or hexagon) shapes connected in a linear, snake-like manner. This structure is often used to model certain types of networks or paths in various fields of study.

A hexagonal snake graph is formed by connecting multiple hexagonal graphs (each forming a hexagon) in a linear, connected fashion such that each successive hexagon shares one or more vertices (or edges) with the previous one. The resulting structure looks like a snake, where each segment of the snake is a hexagonal shape.

A $kC_6$ snake graph has $5k+1$ vertices and $6k$ edges, where $k$ is the number of blocks of a Hexagonal snake graph. That is every edge of a path $v_0 v_1 v_2 \ldots \ldots v_k$ of size $k$ is replaced by a cycle $C_6$ and $d(v_i v_{i+1}) = 2$.




**Theorem: 3.6**

Hexagonal Snake Graph $kC_6$ admits a $k$ -Power 3 Heronian mean graph.

**Proof:**

Let $G = kC_6$ is a connected graph with $5k + 1$ nodes and $6k$ arcs, where $k$ is the number of blocks of a Hexagonal snake.

Let the vertex set of the graph

$$G = \begin{Bmatrix} v_i \ ; \ i\ varies\ from\ k-1 \\ w_i \quad ; \ \ i\ varies\ from\ 1\ to\ k \\ u_i{}^j \ ; \ \ i\ varies\ from\ 1\ to\ k \ \ ; \ \ i\ varies\ from\ 1\ to\ 3 \end{Bmatrix}$$

Define a function $\varphi : V(kC_6) \rightarrow \{k, k + 1, k + 2, \ldots k + q\}$ by

$$\varphi(\alpha_1) = 1 \ ; \ \varphi(\alpha_i) = k + 2(3i - 2); \ i\ varies\ from\ 2\ to\ n$$

$$\varphi(\beta_1) = 2 \ ; \ \varphi(\beta_{i+1}) = k + 6i \ ; \ i\ varies\ from\ 2\ to\ n$$

$$\varphi(\gamma_i) = k + 3(2i - 3) \ ; \ i\ varies\ from\ 1\ to\ n$$

$$\varphi(\delta_i) = k + 3(2i - 1) \ ; \ i\ varies\ from\ 1\ to\ n$$

$$\varphi(\omega_i) = k + 6i - 1 \ ; \ i\ varies\ from\ 1\ to\ n$$

Induced edges are labeled with

$$\varphi(\alpha_i\beta_i) = k + 6i - 5, \ i\ varies\ from\ 1\ to\ n$$

$$\varphi(\alpha_i\beta_{i+1}) = k + 2(3i - 1), \quad i\ varies\ from\ 1\ to\ n$$

$$\varphi(\beta_i\gamma_i) = k + 2(3i - 2) \ ; \quad i\ varies\ from\ 1\ to\ n$$

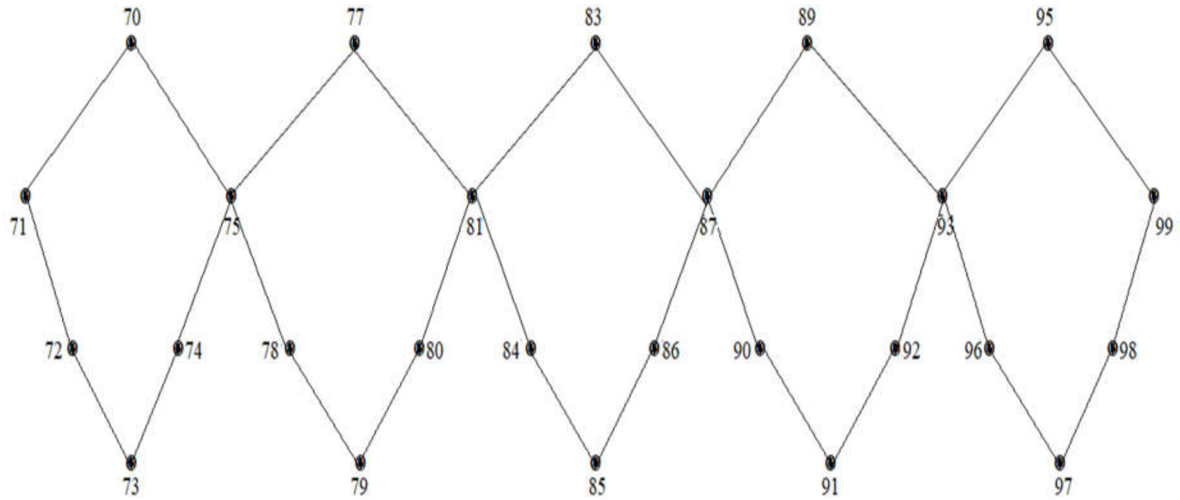
$$\varphi(\gamma_i\delta_i) = k + 3(2i - 1) \ ; \quad i\ varies\ from\ 1\ to\ n$$

$$\varphi(\delta_i\omega_i) = k + 6i - 1 \ ; \quad i\ varies\ from\ 1\ to\ n$$

$$\varphi(\omega_i\beta_{i+1}) = k + 6i \ ; \quad i\ varies\ from\ 1\ to\ n$$

The node and edge labels assigned are distinct. Hence $kC_6$ is a $k$ - Power 3 Heronian Mean graph.

**Illustration 3.7 :** 70 - Power 3 Heronian labelling of $6C_6$




**Conclusion :**

In this article, we introduced the K - Power 3 Mean Heronian Mean Labeling scheme for graphs, a novel approach that combines K – Power 3 labeling with the Heronian Mean. This labeling method provides new avenues for understanding complex relationships between the vertices of a graph, with practical applications in fields such as network design and error correction. By establishing the theoretical framework, examining the mathematical properties, and demonstrating potential applications, we have laid the foundation for further exploration of this labeling technique. Future work can focus on the development of algorithms for efficiently computing the K Power 3 Heronian Mean Labeling labels for large graphs and investigating more specialized cases where the labeling scheme can provide optimal or near-optimal solutions for real-world problems.

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