# Degree Based Some Topological Indices of Pentagonal Quintuple Chains

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**Abstract**: Let $G$ be a simple, undirected, connected graph with vertex set $V(G)$ and edge set $E(G)$. Given $v \in V(G)$, the degree of v is denoted by $d(v)$ and is defined as the number of edges incident with $v$. For the vertices $u, v \in V(G)$, the distance between u and v is denoted by $d(u, v)$ and is defined as the length of the shortest path connecting u and v in G.

In this paper, we make progress to many degree based indices like First and Second zagreb, Randic, Geometric, Arithmetic, Harmonic, Alberston, Atom bond connectivity indices and also obtained closed forms using polynomial of pentagonal quintuple chains.

**Keywords**: Vertex degrees, Zagreb index, Pentagonal Quintuple chain.

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## 1 Introduction

A topological index is a numerical descriptor of the molecular structure derived from the corresponding molecular graph. Many topological indices are widely used for quantitative structure property relationship(QSPR) and quantitative structure activity relationship(QSAR).

Pentagonal chains are important tools in network theory and some chemical applications.In [6], the zagreb indices which from the largest class of topological graph indices have been defined and studied. In [4] some zagreb indices are calculated. In a recent paper, some topological indices of pentagonal chains are calculated [9, 10]. An important and useful variant of pentagonal chains is pentagonal quintuple chains, A pentagonal quintuple chain consisting of k units of pentagons is denoted by $C^5_{5,k}$ and illustrated in Figure

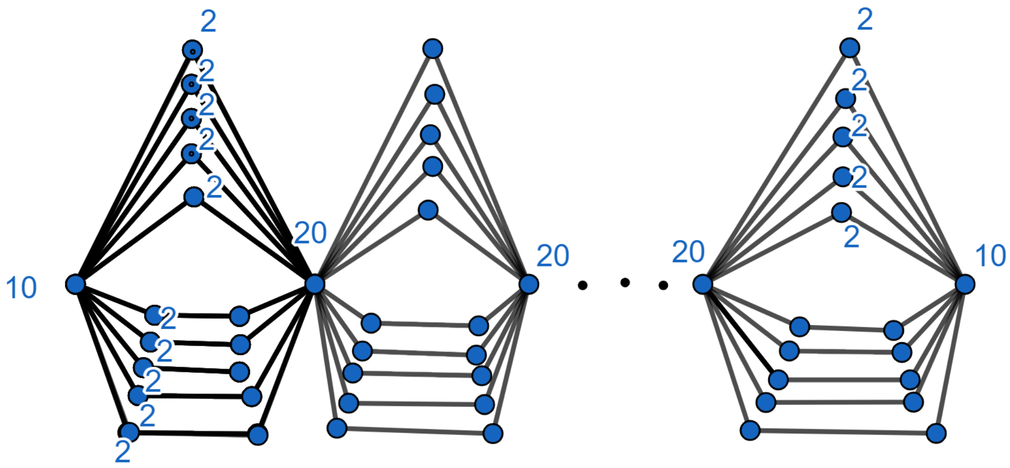


Figure:Pentagonal quintuple chain

The degree sequence of $C^5_{5,k}$ consisting of the vertex degree is $D\left(C^5_{5,k}\right) = \left\{2^{15k}.10^2.20^{k-1}\right\}$

Similarly, the edge partition table of $C^5_{5,k}$ is as in Table.

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| $(d(u), d(v))$ | Number of vertices $(u, v)$ |
|---|---|
| (2,2) | 5k |
| (2,10) | 20 |
| (2,20) | 20k-20 |

The omega invariant,[3], of the connected graph $C_{5,k}^5$ is

$\Omega\left(C_{5,k}^5\right) = 8.2 + 18(k-1) = 18k - 2$

Hence the number of faces of $C_{5,k}^5$ would be $\dfrac{\Omega\left(C_{5,k}^5\right)}{2} + 1 = 9k$

In following sections, we shall make our proofs by means of this degree sequence and edge partition table.

## 2 Additive topological indices of $C_{5,k}^5$

Most of the topological indices are defined in terms of vertex degrees. Each such degree based index consists of some mathematical formula. This formula frequently contains a sum or product over the vertices or edges of the graph. In this section, we calculate some additive topological indices of the pentagonal quintuple chain.

The first and second zagreb index is defined as

$M_1(G) = \sum_{uv \in E(G)} d(u) + d(v)$ and $M_2(G) = \sum_{uv \in E(G)} d(u)d(v)$

These indices were introduced by Gutman and Trinajestic.

The inverse sum index is defined by

$ISI(G) = \sum_{uv \in E(G)} \dfrac{d(u)d(v)}{d(u)+d(v)}$

The Sigma index is an important irregularity measure defined by

$$\sigma(G) = \sum_{uv \in E(G)} (d(u) - d(v))^2.$$

The Harmonic index is defined by: $H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}$

The Atom bond connectivity index ($ABC$) is defined by:

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d(u) + d(v) - 2}{d(u)d(v)}}$$

The Geometric-arithmetic index is defined as:

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d(u)d(v)}}{d(u) + d(v)}$$

The Augmented Zagreb index is :

$$AZ(G) = \sum_{uv \in E(G)} \left( \frac{d(u)d(v)}{d(u) + d(v) - 2} \right)^3$$

The irregularity index is the Albertson index is defined by the sum of absolute values of all the differences between degrees of pairs of vertices forming an edge:

$$Alb(G) = \sum_{uv \in E(G)} |d(u) - d(v)|$$

The Randi´c index is defined as:

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d(u)d(v)}}$$

The Reciprocal Randi´c index is defined similarly to Randi´c index:

$RR(G) = \sum_{uv \in E(G)} \sqrt{d(u)d(v)}$

Now we present our results on above definitions :

**Theorem 2.1.** *Some additive degree-based topological indices of the pentagonal Quintuple chain* $C^5_{5,k}$ *are as follows:*

$$M_1(G) = 460k - 200$$
$$M_2(G) = 820k - 400$$
$$ISI(G) = \frac{1365k - 100}{33}$$
$$\sigma(G) = 6480k - 5200$$
$$H(G) = \frac{285k + 100}{66}$$
$$ABC(G) = \frac{25k}{\sqrt{2}}$$
$$GA(G) = 5k + \frac{20}{3}\sqrt{5} + 40(k-1).\frac{\sqrt{10}}{11}$$
$$AZ(G) = 200k$$
$$Alb(G) = 360k - 200$$
$$R(G) = \frac{5k}{2} + \sqrt{20} + \sqrt{10}(k-1)$$
$$RR(G) = 10k + 40\sqrt{5} + 20(2k-1)\sqrt{10}$$

***Proof:*** *Let* $G = C^5_{5,k}$ *be a pentagonal Quintuple chain graph with* $n = 16k + 1$ *vertices and* $m = 25k$ *edges. The Proofs are as follows.*

$M_1(G) = 5k(2+2) + 20(2+10) + (20k-20)(2+20)$

$= 460k - 200.$

*Next we find second additive Zagreb index.*

$M_2(G) = 5k(2*2) + 20(2*10) + (20k-20)(2*20)$

$= 820k - 400$

*in a similar manner we calculated all other indices.*

## 3 Multiplicative topological indices of $C_{5,k}^5$

*In this section, we calculate some multiplicative topological indices of pentagonal quintuple chain $C_{5,k}^5$. These multiplicative versions of the indices are obtained by replacing the sum sign with a product sign. These Indices are listed below:*

$$PM_1(G) = \prod_{uv \in E(G)} d(u) + d(v) \text{ and } PM_2(G) = \prod_{uv \in E(G)} d(u)d(v)$$

*these are called the Multiple first and second Zagreb indices. Similarly the multiplicative forgotten index, sometimes named as the multiplicative third Zagreb index is defined by:*

$$\prod_3(G) = \prod_{uv \in E(G)} d^2(u) + d^2(v)$$

*The Geometric-arithmetic multiplicative index is defined as:*

$$GA\prod(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d(u)d(v)}}{d(u) + d(v)}$$

*The first and second multiplicative hyper Zagreb indices are:*

$$H\prod_1(G) = \prod_{uv \in E(G)} (d(u) + d(v))^2$$

*and*

$$H\prod_2(G) = \prod_{uv \in E(G)} (d(u)d(v))^2$$

*Multiplicative General sum connectivity index is given by*

$$H_\alpha(G) = \prod_{uv \in E(G)} (d(u) + d(v))^\alpha$$

*The Multiplicative Randić index is defined by*

$R\prod(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d(u)d(v)}}$

*The multiplicative sum connectivity index is defined by*

$\chi\prod(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d(u)+d(v)}}$

*The multiplicative atom bond connectivity index is defined by*

$ABC\prod(G) = \prod_{uv\in E(G)} \sqrt{\frac{d(u)+d(v)-2}{d(u)d(v)}}$

*then we have the following result:*

**Theorem 3.1.** *Some multiplicative degree-based topological indices for the the pentagonal Quintuple chain* $C^5_{5,k}$ *are as follows:*

$\prod_1(G) = 4^{5k}.12^{20}.22^{20(k-1)}$

$\prod_2(G) = 2^{(70k-20)}.5^{20k}$

$\prod_3(G) = 8^{5k}.104^{20}.404^{20(k-1)}$

$GA\prod(G) = 5^{10k}.3^{-20}.\left(\frac{2}{11}\right)^{20(k-1)}$

$H\prod_1(G) = 2^{(60k+40)}.3^{40}.11^{40(k-1)}$

$H\prod_2(G) = 2^{(140k-40)}.5^{40k}$

$H\prod_\alpha(G) = 2^{10(3k+2)\alpha} * 3^{20\alpha}.11^{20(k-1)\alpha}$

$R\prod(G) = 2^{(10-35k)}.5^{-10k}$

$\chi\prod(G) = 2^{(-10-15k)}.3^{-10}.11^{10(1-k)}$

$ABC\prod(G) = \left(\frac{1}{\sqrt{2}}\right)^{25k}$

***Proof:*** *Let* $G = C^5_{5,k}$ *be a pentagonal Quintuple chain graph with* $n = 16k + 1$ *vertices and* $m = 25k$ *edges. The Proofs.*

*We calculate the multiplicative topological indices of* $C^5_{5,k}$ *by using existing multiplicative topological indices are mentioned above and using the degree sequence of table 1. We get*

$\prod_1(G) = (2+2)^{5k} * (2+10)^{20} * (2+20)^{20(k-1)} = 4^{5k}.12^{20}.22^{20(k-1)}$

$\prod_2(G) = (2*2)^{5k} * (2*10)^{20} * (2*20)^{20(k-1)} = 2^{(70k-20)}.5^{20k}$

*Remaining results have been proven as above*

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