Replication of Graph Elements in Cubic Heronian Mean Graphs

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Abstract:

A Cubic Heronian Mean Graph (CHMG) is a specific type of labeled graph where the weights of nodes and links are determined using the cubic Heronian mean function, which creates a clear labeling scheme. This paper examines the replication of graph elements such as nodes, links and substructures in CHMGs, looking into the necessary conditions for these replications and their effects on graph properties. We present new constructions of CHMGs using replication techniques, analyze the structural consequences, and establish mathematical limits to preserve their Heronian mean properties. Our results enhance the broader understanding of graph labelings by broadening the range of graphs that can support cubic Heronian mean labeling.

Key words: Labeling, Cubic Heronian Mean Labeling, Replication of a nodes and edges.

1. Introduction:

Graph theory plays a fundamental role in various mathematical and real-world applications, ranging from computer science and network analysis to biological systems and cryptography. Among the numerous graph labeling techniques studied, Heronian mean labeling has emerged as a significant approach due to its unique mathematical properties and applicability in structured graph analysis. The study of Heronian mean graphs, particularly cubic Heronian mean graphs, extends the field by incorporating cubic Heronian mean functions in labeling schemes, leading to intriguing combinatorial properties.

Graph element replication specifically, the replication of nodes and links is an essential concept in graph transformation and expansion. Understanding how these elements can be replicated while preserving specific labeling properties allows for constructing larger, more complex networks while maintaining underlying mathematical consistencies. In this study, we explore the replication of elements in cubic Heronian mean graphs, analyzing the conditions under which such replications retain the Heronian mean properties.

This paper systematically investigates the structural and numerical implications of element replication in cubic Heronian mean graphs. We propose novel construction methods that facilitate the extension of these graphs while ensuring the integrity of their labeling schemes. Additionally, we derive necessary conditions for preserving Heronian mean properties in replicated structures and discuss their potential applications in theoretical and applied graph theory.

2. Basic Definitions and Results

In this section, we outline the essential concepts needed to understand cubic Heronian mean graphs and their characteristics. We will cover graph labeling techniques, the Heronian mean function, and the necessary mathematical foundations that will support our later discussions.

Graph Replication: This refers to the process of copying vertices (nodes) and edges (links) in a graph while keeping its structural and labeling properties intact.

Vertex Replication: This involves adding new vertices to the graph that maintain the same connectivity properties as the original vertices.

Edge Replication: This is the process of duplicating edges while ensuring that the overall connectivity and labeling rules of the graph remain unchanged.

Cubic Heronian Mean Labeling:

Let a graph G consist of number p nodes, and q edges; then it is termed Cubic Heronian graph (CHMG) if the distinct integers 1 to q+1 can label all the vertices such that for every edge e connecting vertices u and v, edge label that is defined as a function 'f' as $f(\alpha)$, for vertex α and $f(\beta)$ for vertex β across

$$\varphi(e=\alpha\beta)=\left\lceil\left(\frac{\varphi(\alpha)^3+(\varphi(\alpha)\varphi(\beta))^{\frac{3}{2}}+\varphi(\beta)^3}{3}\right)^{\frac{1}{3}}\right\rceil \ or \ \left\lfloor\left(\frac{\varphi(\alpha)^3+(\varphi(\alpha)\varphi(\beta))^{\frac{3}{2}}+\varphi(\beta)^3}{3}\right)^{\frac{1}{3}}\right\rfloor$$

then the edge labels as also different. In this case, a labeling function φ is known as a Cubic Heronian Mean labeling of G. It is a labeling scheme that assigns values of vertices and edges using the cubic Heronian mean characteristic-functional equivalent labels.

Structural Integrity in Replication: These are the conditions that must be met for the replication of graph elements to preserve the original labeling characteristics and structural properties of the graph.

3. Major Outcomes

Proposition 3.1 For n less than 5, the graph formed by replicating each vertex v_i in the complete bipartite graph is a Cubic Heronian Mean Graphs.

Proof:

Let u be the apex vertex and $v_1, v_2, ..., v_n$ be the consecutive vertices of $K_{1,n}$. Consider G as the graph obtained by replicating a vertex vi with a vertex vj in $K_{1,n}$. The resulting graph G will consist of 2n vertices and 2n edges.

Let G be a graph with vertex set V(G). Define a function φ as a mapping that assigns a unique integer from the set {1, 2, ..., 2n} to each vertex in V(G) in the following manner.

φ (u) = 1

φ (v_i) = i + 1; ; i *varies from* 1 *to* n

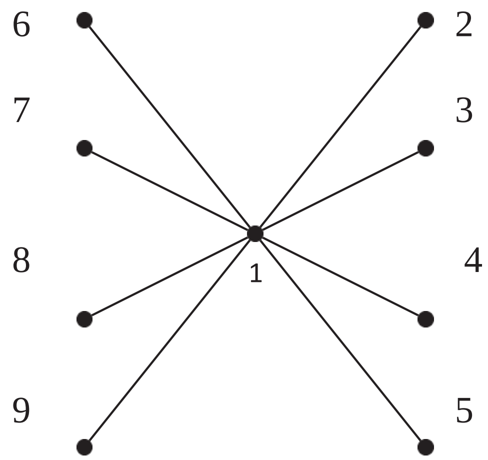
$\varphi(v_i{}') =$ n + i + 1; ; i *varies from* 1 *to* n

Edges are labeled with φ (uv_i) = i; ; i *varies from* 1 *to* n

$\varphi(uv_i{}') =$ n + i; $v_i{}'$

In light of the labeling pattern described above, we obtain distinct edge labels ranging from 1 to 2n2n2n. Therefore, the graph resulting from the replication of each vertex will have these unique edge labels in $K_{1,n}$ is Cubic Heronian Mean graph for n less than 5

Illustration 3.2 The graph obtained by replicating every vertex in K1,n, along with its corresponding Cubic Heronian Mean labeling, is illustrated below



This graph demonstrates the Cubic Heronian Mean (CHM) labeling property, where each edge label is the Heronian mean of the labels of its incident vertices.

Proposition 3.3 The graph obtained by replication of an edge in $K_{1,n}$ is a Cubic heronianMean graph for n < 5.

Proof:

The graph you mentioned is a variation of the complete bipartite graph K1,n, where the vertex vi is duplicated by another vertex vj. This results in a new graph G that has 2n vertices and 2n edges.

Define $\varphi : V(G) \rightarrow 1, 2, \ldots, 2n$ as follows.

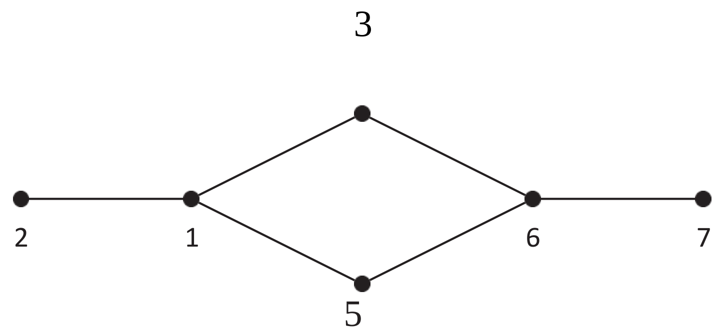
$\varphi(u) = 1$

$\varphi(v_i) = i + 1;\ i = 1,2$

$\varphi(v_i) = i + 2;\ i = 3,4$

The labeling pattern yields distinct edge labels from 1 to 2n. Therefore, the graph obtained by replicating every vertex in K1,n is a Cubic Heronian Mean (CHM) graph for n < 5.

Illustraction 3.4 The graph created by duplicating an edge in K1,3 along with its Cubic Heronian Mean labeling is displayed below.



Proposition 3.5. Replicating the apex vertex by an edge in $K_{1,n}$ yields a graph that exhibits Cubic Heronian Mean graph properties for n le than or equal to 6.

Proof. Let u be the apex vertex and $v_1v_2, \ldots, v_n$ be the consecutive vertices of $K_{1,n}$. Let G be the graph obtaind by replication of apex vertex by an edge in $K_{1,n}$. The resultant graph G will have n + 2 nodes and n + 3 links.

Define $\varphi: V(G) \rightarrow \{1, 2, \ldots, n+3\}$ as follows.

$\varphi(u) = 1$

$\varphi(v_i) = i+1;\ 1 \leq i \leq n$

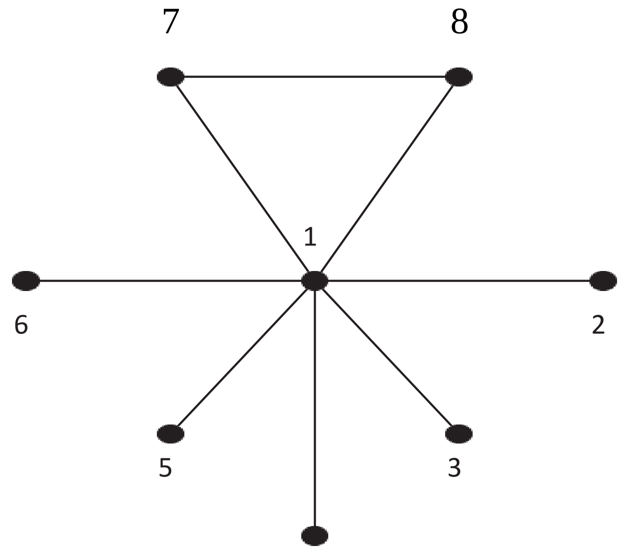
$\varphi(u_i') = n + i + 1\ ; i = 1,2$

Edges are labeled with $\varphi(uv_i) = i\ ;\ \ i\ varies\ from\ 1\ to\ n$

$$\varphi(uv_i') = n + i\ ;\ \ i\ varies\ from\ 1\ to\ n$$

In light of the labeling pattern mentioned above, we have distinct edge labels from {1, 2,..., n +3}. Therefore, the graph created by replicating the apex vertex in K1,n is a Cubic Heronian Mean graph for n ≤ 6.

Illustraction 3.6 The graph resulting from replicating the apex vertex by an edge in $K_{1,5}$ along with its corresponding Cubic Heronian mean labeling, is illustrated below



Proposition 3.7. The graph obtained by replication of a vertex other than apex vertex by an link in $K_{1,n}$ is a Cubic Heronian Mean graph for n le than or equal to 8.

Proof. Consider a star graph $K_{1,n}$ with apex vertex u and consecutive vertices vi ; $i = 1\ to\ n$ .Replicate the apex vertex u by an edge in $K_{1,n}$ to form a new graph G with $n + 2$ nodes and $n + 3$ links.

Define $\varphi : V(G) \rightarrow \{1, 2, \ldots, n + 4\}$ as follows.

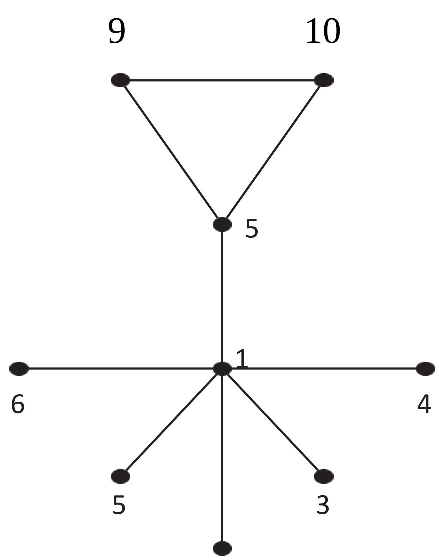
$\varphi(u) = 1$

$\varphi(vi) = i + 1;\ i\ varies\ from\ 1\ to\ n$

$\varphi(v_j') = n + 1$; $\varphi(v_j') = n + 4$

In light of the labeling pattern mentioned above, we have unique edge labels ranging from {1, 2,..., n + 3}. Therefore, the graph formed by replicating a vertex through an edge in K1,n is a cubic Heronian mean graph for n ≤ 8.

Illustraction 3.8 The graph created by replicating a vertex through an edge in $K_{1,n}$, along with its Cubic Heronian mean labeling, is illustrated below.



Proposition 3.9. The graph obtained by replication of n edge by a vertex in $K_{1,n}$ is a Cubic Heronian Mean graph for $n \leq 8$.

Proof.

Let u be the apex vertex and $v_1v_2, \ldots, v_n$ be the consecutive vertices of $K_{1,n}$. Let G be the graph obtaind by duplication of an edge uv_n by a vertex $K_{1,n}$ by a vertex v_n'. The resultant graph will have $n+1$ nodes and $n+2$ links.

Define $\varphi : V(G) \to \{1, 2, \ldots, n + 3\}$ as follows.

$\varphi(u) = 1$

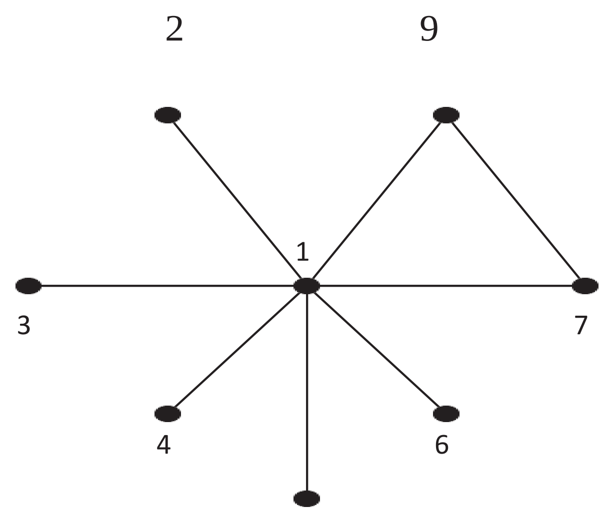
$\varphi(vi) = i + 1; i\ varies\ from\ 1\ to\ n$

$\varphi(v_n') = n + 3$

In light of the labeling pattern mentioned above, we have unique edge labels ranging from {1, 2,..., n + }. Therefore, the graph formed by replicating an edge by a vertex in $K_{1,n}$ is Cubic Heronian Mean graph for $n \leq 8$.

Illustraction 3.10 The graph created by replicating a vertex through a verte in

$K_{1,n}$, along with its Cubic Heronian mean labeling, is illustrated below.



Proposition 3.11. The graph obtained by replication of every edge by a vertex in $K_{1,n}$ is a Cubic Heronian Mean graph for $n \leq 4$.

Proof. Let u be the apex vertex and $v_1 v_2, \ldots, v_n$ be the consecutive vertices of $K_{1,n}$. Let G be the graph obtained by replication of edges uv_i by a vertex v_i' in $K_{1,n.}$ The resultant graph G will have $2n+1$ nodes and $3n$ links.

Define $\varphi : V(G) \rightarrow \{1, 2, \ldots, n + 3\}$ as follows.

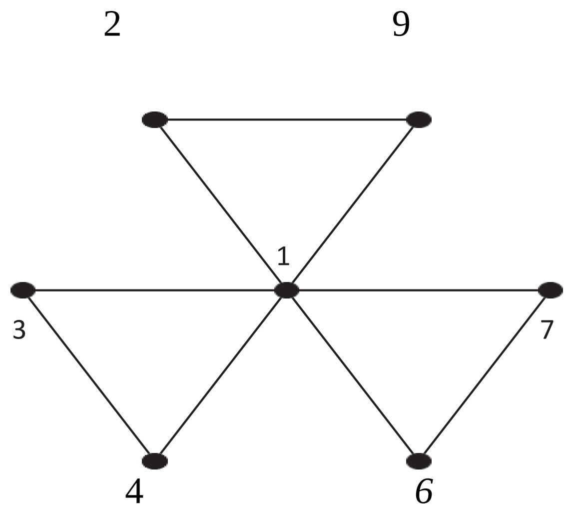
$\varphi(u) = 3$; $\varphi(v_1) = 2$

$\varphi(vi) = 3i + 1;$ i varies from 2 to n

$\varphi(v_1') = 1$; $\varphi(v_i') = 3i$; i varies from 2 to n

In view of the above labeling pattern we have distint edge labels from 1, 2,…, $3n + 1$. Hence the graph obtained by replication of every edge by a vertex in $K_{1,n}$ is Cubic Heronian Mean graph for $n \leq 8$.

Illustraction 3.12 The graph created by replicating a vertex through every edge in K $_{1,n}$, along with its Cubic Heronian mean labeling, is illustrated below.



Conclusion :

In this paper we attempted to find out how repeat graph entities-vertices and edges-will affect certain properties of Cubic Heronian Mean Graphs. After thorough consideration of this matter, we found that replication in the form of graph elements induces not only changes in structural properties of cubic graphs but also affects the mean values of the graph when the mean Cubic Heronian is considered. Thus, the paper provides a base for seeing how replication operation affects certain forms of graph mostly when it comes to the cubic mean.

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