Fuzzy Range Labeling Of Certain Fuzzy Anti-Magic Graphs

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ABSTRACT

The aim of this article is to check the fuzzy range labeling on some other labeled fuzzy graphs. For that, the authors implemented this plan on fuzzy anti-magic labeling graphs. This work proved that path and butterfly graphs are fuzzy range graph.

Keywords : *Range labeling, Fuzzy range labeling, Fuzzy anti-magic labeling, Path graph, Butterfly graph.*

1. INTRODUCTION

The graph labeling concept was originated by Rosa [1] in 1967. Refer to Gallian's [2] survey for more ideas about graph labeling. Among them, one of the recently proposed topics is range labeling graph by R. Jahir Hussain and J. Senthamizh Selvan [3].

Due to the world's unpredictability, fuzzy graphs are more helpful than crisp graphs. But both structures are comparable. There are many different areas of fuzzy graph theory that are able to be researched. Especially, fuzzy graph labeling, an evolving area of graph theory research, can be found in plenty of real-world situations, such as biological systems, missile guidance, pulse radar, social networks like computer networks, communication networks, etc. It has been gaining significant attention in recent years due to its impressive practical applications. In particular, it is used in the networking model when data inconsistencies arise. For instance, assigning unique numbers to user terminals in a communication network ensures that the numbers represent the connecting path between them. Here, fuzzy magic and anti-magic labeling help identify the pair of terminals connected by a specific path number.

A. Nagoor Gani and D. Subahashini [4] explained the concepts of fuzzy labeling and fuzzy magic labeling as well as discussed their properties. N. Hartsfield and G. Ringel [5] introduced the anti-magic labeling graph in 1990. K. Ameenal Bibi and M. Devi [6] established fuzzy anti-magic graphs. R. Jebesty Shajila and S. Vimala [7] discussed fuzzy graceful labeling and fuzzy anti-magic labeling of path and butterfly graphs.

The notion of Fuzzy Range Labeling was initiated by S. Ramya *et al.* [8],[9]. This technique has made progress by them on several graphs in assorted types. This article offers further visions of fuzzy range labeling. We use only finite, simple and undirected graphs.

2. PRELIMINARIES

Definition 2.1 [3]: Range Labeling Graph

Let $G=(V,E)$ be a graph with n vertices. A bijection on $f:V\rightarrow\{1,2,...,n\}$ is called a Range Labeling if for each edge E is distinct and E is defined by

$$f^{*}(E)=Maximum\,value\left(v_k,v_{k+1}\right)-Minimum\,value\left(v_k,v_{k+1}\right).$$

If a graph G admits range labeling, we say G is a range graph.

Definition 2.2 [4]: Fuzzy Labeling Graph

The bijective functions $\sigma:V\rightarrow[0,1]$ and $\mu:V\times V\rightarrow[0,1]$ such that the membership values of edges and vertices are distinct and $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$, is called fuzzy labeling. A graph which admits fuzzy labeling is called a fuzzy labeling graph.

Definition 2.3 [8]: Fuzzy Range Labeling Graph

The bijective functions $\sigma:V\rightarrow[0,1]$ and $\mu:V\times V\rightarrow[0,1]$ subject to the conditions $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ and $\mu(u,v)=Max.\,value(\sigma(u),\sigma(v))-Min.\,value(\sigma(u),\sigma(v))$; for all $u,v\in V$, such that the membership values of edges and vertices are distinct, is called fuzzy range labeling. A graph that admits fuzzy range labeling is called fuzzy range graph.

Definition 2.4 [10]: Fuzzy Anti-Magic Labeling Graph

If $\sigma(u)+\mu(u,v)+\sigma(v)$; $\forall u,v\in V$ are all distinct then the fuzzy graph is called fuzzy edge anti-magic labeling graph.

A fuzzy graph is called fuzzy vertex anti-magic labeling graph in which for any two vertices u and v, the sum of the labels on edges incident to u distinct from the sum of the labels on edges incident to v.

Definition 2.5 [11]: Path Graph

The path graph P_n is a tree with two vertices (terminal) of degree 1 and the other $n-2$ vertices of degree 2. All of its vertices and edges lie on a single straight line.

Definition 2.6 [12]: Butterfly Graph

A butterfly graph is a double shell with exactly two pendent edges at the apex and each shell has any order.

3. RESULTS

Theorem : 3.1

All fuzzy anti-magic path graph $\mathbb{P}_m$ admits fuzzy range labeling.

Proof:

Let us take the vertices and edges of fuzzy anti-magic path graph $\mathbb{P}_m$ as $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_{m-1}$.

Let $s \to (0,1]$ so that one can prefer $s = \begin{cases} 0.01 \ ; \text{ if } \ m \le 7 \\ 0.001 \ ; \text{ if } \ m > 7 \end{cases}$

Now, for $3 \le m \le 7$, based on the edges common difference, fix the labels of $\mathbb{P}_m$ as follows:

Case (i):

While the edge difference is $2s$, $\mathbb{P}_m$ concedes fuzzy range labeling for every $3 \le m \le 7$.

Here $\beta(a_{r+1}\, a_{r+2}) - \beta(a_r\, a_{r+1}) = 2s$; $s = 0.01$ and $r = 1, 2, \ldots, m-2$

and $\beta(a_r\, a_{r+1}) = Max(\alpha(a_r), \alpha(a_{r+1})) - Min(\alpha(a_r), \alpha(a_{r+1})) = 2rs$; $r = 1, 2, \ldots, m-1$.

Also, $\alpha(a_r) + \beta(a_r\, a_{r+1}) + \alpha(a_{r+1})$ is distinct for every $r = 1, 2, \ldots, m-1$.

Case (ii):

While the edge difference is $3s$, $\mathbb{P}_m$ concedes fuzzy range labeling for every $3 \le m \le 6$.

Here $\beta(a_{r+1}\, a_{r+2}) - \beta(a_r\, a_{r+1}) = 3s$; $s = 0.01$ and $r = 1, 2, \ldots, m-2$

and $\beta(a_r\, a_{r+1}) = Max(\alpha(a_r), \alpha(a_{r+1})) - Min(\alpha(a_r), \alpha(a_{r+1})) = 3rs$; $r = 1, 2, \ldots, m-1$.

Example : 3.1.1

Fig. 1. Fuzzy Range Labeling for Fuzzy Anti-Magic Path Graph $\mathbb{P}_6$

Case (iii):

While the edge difference is $4s$, $\mathbb{P}_m$ concedes fuzzy range labeling for every $3 \le m \le 5$.

Here $\beta(a_{r+1}\, a_{r+2}) - \beta(a_r\, a_{r+1}) = 4s$; $s = 0.01$ and $r = 1, 2, \ldots, m-2$

and $\beta(a_r\, a_{r+1}) = Max(\alpha(a_r), \alpha(a_{r+1})) - Min(\alpha(a_r), \alpha(a_{r+1})) = 4rs$; $r = 1, 2, \ldots, m-1$.

Case (iv):

While the edge difference is $5s$ or $6s$ or $7s$, $\mathbb{P}_m$ concedes fuzzy range labeling for every $3 \le m \le 4$.

Case (v):

While the edge difference is $8s$ or $9s$ or $10s$ or $11s$ or $12s$, $\mathbb{P}_m$ concedes fuzzy range labeling for $m=3$.

It is verified that $\alpha(a_r)+\beta(a_r\ a_{r+1})+\alpha(a_{r+1})$ is distinct in all the above cases for every $r=1,2,...,m-1$.

Hence, all fuzzy anti-magic path graph $\mathbb{P}_m$ is fuzzy range graph for every $3\le m\le 7$.

Theorem : 3.2

The fuzzy anti-magic butterfly graph $\mathbb{B}_m$ (except the apex region) admits fuzzy range labeling.

Proof:

Let the butterfly graph $\mathbb{B}_m$ with m vertices and e edges.

In $\mathbb{B}_m$, when $m=7$, $e=8$; when $m=9$, $e=12$; and so on.

Let $s\to(0,1]$. Consider that $s=\begin{cases}0.01\ ; & \text{if } m=7 \text{ or } 9\\ 0.001\ ; & \text{otherwise}\end{cases}$

Let us think of as all the edges of $\mathbb{B}_m$ incident to the common vertex a_1 (apex).

Now, we prove this result by giving $\mathbb{B}_9$ as example.

Here, $\beta(a_1,a_2)=Max(\alpha(a_1),\alpha(a_2))-Min(\alpha(a_1),\alpha(a_2))=0.01$

$\beta(a_1,a_3)=Max(\alpha(a_1),\alpha(a_3))-Min(\alpha(a_1),\alpha(a_3))=0.03$

$\beta(a_1,a_4)=Max(\alpha(a_1),\alpha(a_4))-Min(\alpha(a_1),\alpha(a_4))=0.06$

$\beta(a_1,a_5)=Max(\alpha(a_1),\alpha(a_5))-Min(\alpha(a_1),\alpha(a_5))=0.10$

$\beta(a_1,a_6)=Max(\alpha(a_1),\alpha(a_6))-Min(\alpha(a_1),\alpha(a_6))=0.15$

$\beta(a_1,a_7)=Max(\alpha(a_1),\alpha(a_7))-Min(\alpha(a_1),\alpha(a_7))=0.21$

$$\beta(a_1,a_8)=Max(\alpha(a_1),\alpha(a_8))-Min(\alpha(a_1),\alpha(a_8))=0.28$$

$$\beta(a_1,a_9)=Max(\alpha(a_1),\alpha(a_9))-Min(\alpha(a_1),\alpha(a_9))=0.36$$

Further, $\beta(a_4,a_5)=Max(\alpha(a_4),\alpha(a_5))-Min(\alpha(a_4),\alpha(a_5))=0.04$

$$\beta(a_5,a_6)=Max(\alpha(a_5),\alpha(a_6))-Min(\alpha(a_5),\alpha(a_6))=0.05$$

$$\beta(a_7,a_8)=Max(\alpha(a_7),\alpha(a_8))-Min(\alpha(a_7),\alpha(a_8))=0.07$$

$$\beta(a_8,a_9)=Max(\alpha(a_8),\alpha(a_9))-Min(\alpha(a_8),\alpha(a_9))=0.08$$

It is noted that all $\beta(xy)+\alpha(y)+\beta(yz)$; $\forall x,y,z\in V$ is distinct and also $\mathbb{B}_9$ satisfies fuzzy range labeling conditions.

Example : 3.2.1

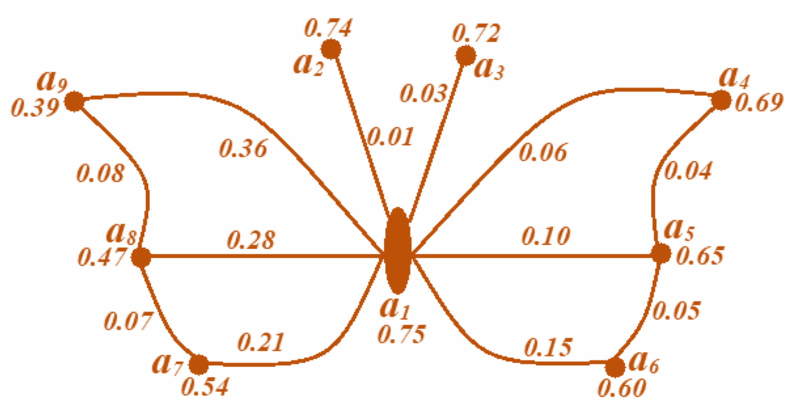

Fig. 2. Fuzzy Range Labeling for Fuzzy Anti-Magic Butterfly Graph $\mathbb{B}_9$

Thus, the fuzzy anti-magic butterfly graph $\mathbb{B}_m$ (except the apex region) is fuzzy range graph.

4. CONCLUSION

We deduce this fact-finding by showing that the fuzzy anti-magic labeled path and butterfly graphs are fuzzy range graph. We think out to perform this research on some more labeled graphs in the future.

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