PENDANT DOMINATION OF n-SUNLET GRAPH

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Abstract

Let $G = (V, E)$ be any graph. A dominating set S in G is said to be a pendant dominating set if induced sub graph of S contains at least one pendant vertex. The least cardinality of a pendant dominating set in G is said to be a pendant domination number of G and is denoted by $\gamma_{pe}(G)$. A pendant dominating set S of G is said to be connected pendant dominating set if the induced sub graph is connected and the connected pendant domination number is denoted by $\gamma_{cpe}(G)$. A pendant dominating set S of G is said to be split pendant dominating set if the induced sub graph is disconnected and and the split pendant domination number is denoted by $\gamma_{spe}(G)$. In this article, we determine the pendant domination number, split pendant domination number and connected domination number of n - Sunlet graph S_n.

Keywords : Dominating Set, Domination number,Sunlet Graph, Pendant Dominating set, Split pendant dominating set and Connected pendant dominating set.

1 Introduction

Domination is the most important and vastly growing research area in the field of graph theory. The study of domination in graph theory is fastest growing area and it came as a result of study of games such as game of chess where the goal is to dominate various squares of a chessboard by certain chess pieces. The concept of domination was used by De Jaenisch in 1862 while studying the problems of determining the minimum number of queens to dominate chessboard. Berge defined the concept of domination number of graph. But the fastest growth in study of dominating set in graph theory began in 1960. Later the concept of domination set and domination number was used by Ore in 1962, Cockayne and Hedetniemi in 1977.

The concept of domination has wide range of application in Graph theory. The concept of domination enables us to find the shortest or longest distance between any two points or places. Domination is also used in the field of land surveying, Electrical networks, Networking, Routing problems, nuclear plants problem, Modeling problems, Coding theory etc. In this article, we discuss about the pendant domination number, split pendant domination number and connected domination number of n-Sunlet graph S_n.

In the entire paper we consider the finite, undirected, simple and non-trivial graph. The n-sunlet graph of vertices $2n$ is obtained by attaching n - pendant edges to the cycle graph C_n and it is denoted by S_n

2 Definitions

Definition 2.1. *A subset S of the set of vertices $V(G)$ is said to be Dominating set of G if every vertex in $V-S$ is adjacent to at least one vertex in S. The number of vertices of a minimum dominating set of a graph G is called Domination number of G and it is denoted by $\gamma_{pe}(G)$*

Definition 2.2. *Let $G=(V,E)$ be any graph. A dominating set S in G is said to be a pendant dominating set if induced sub graph of S contains at least one pendant vertex. The least cardinality of a pendant dominating set in G is said to be a pendant domination number of G and is denoted by $\gamma_{pe}(G)$.*

Definition 2.3. *The n- sunlet graph is a graph on $2n$ vertices is obtained by attaching n - pendant edges to the cycle graph C_n and it is denoted by S_n.*

Definition 2.4. *The pendant dominating set S of a graph G is said to be connected pendant dominating set of G if induced sub graph $<V-S>$ is connected. The minimum cardinality of a connected pendant dominating set is called connected pendant dominating number and it is denoted by $\gamma_{cpe}(G)$.*

Definition 2.5. *The pendant dominating set S of a graph G is said to be split pendant dominating set of G if induced sub graph $<V-S>$ is disconnected. The minimum cardinality of a split pendant dominating set is called split pendant dominating number and it is denoted by $\gamma_{spe}(G)$.*

Definition 2.6. *The pendant dominating set S of a graph G is said to be non-split pendant dominating set of G if induced sub graph $<V-S>$ is connected. The minimum cardinality of a non-split pendant dominating set is called non-split pendant dominating number and it is denoted by $\gamma_{nspe}(G)$.*

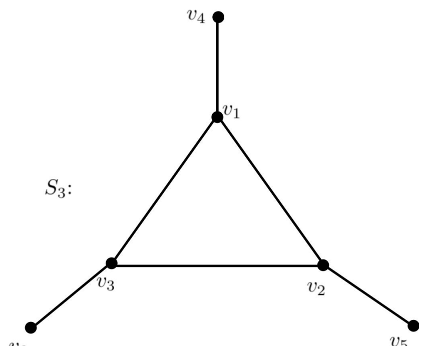
Remarks:
(i) Number of vertices in n - sunlet graph S_n is $p=2n$
(ii) Number of edges in n - sunlet graph S_n is $q=2n$
(iii) Maximum degree in n - sunlet graph S_n is $\Delta=3$
(iv) Minimum degree in n - sunlet graph S_n is $\delta=1$

3 Pendant domination of n - Sunlet graph

Proposition 3.1. [9] *The domination number of a 3-sunlet graph is 3 i.e. $\gamma(S_3)=3$*

Theorem 3.1. *The pendant domination number of a 3-sunlet graph is* 3

$$\gamma_{pe}(S_3)=3$$



3 - Sunlet graph

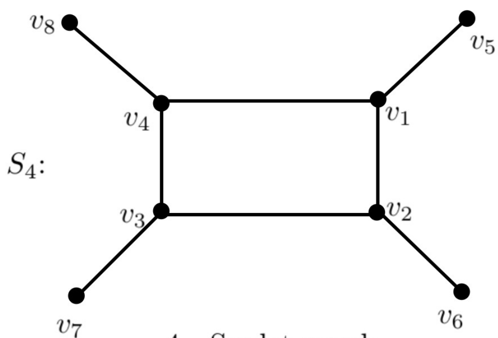
Proof. Let $S = \{v_1, v_2, v_3\}$ be a vertex set and $\{v_4, v_5, v_6\}$ are the pendant vertices whose degree is 1. Since all the vertices of S_3 is adjacent to atleast one pendant vertex of S, thus S is a pendant dominating set of S_3.
To prove that S is a minimal pendant dominating set, let us consider $S' = S - \{v_2\} = \{v_1, v_3\}$ in which the pendant vertex $\{v_5\}$ is not dominated by any of the vertices of S'. Thus S' is not a pendant dominating set.Therefore S is the minimal pendant dominating set and hence the pendant domination number of 3-sunlet graph is 3 i.e $\gamma_{pe}(S_3) = 3$

□

Proposition 3.2. [9] *The domination number of a 4-sunlet graph is 3 i.e.* $\gamma(S_4) = 4$

Theorem 3.2. *The pendant domination number of a 4-sunlet graph is* 4

$$\gamma_{pe}(S_4) = 4$$

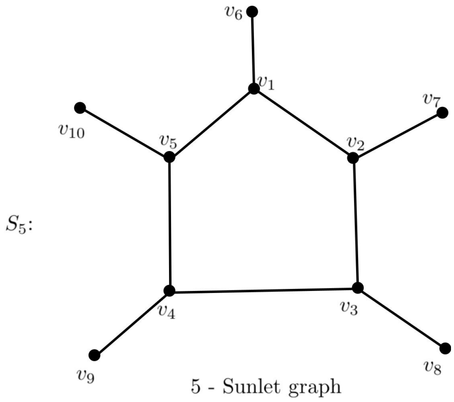


4 - Sunlet graph

Proof. Let $S = \{v_1, v_2, v_3, v_4\}$ be a vertex set and $\{v_5, v_6, v_7, v_8\}$ are the pendant vertices whose degree is 1. Since all the vertices of S_4 is adjacent to atleast one pendant vertex of S, thus S is a pendant dominating set of S_4.
To prove that S is a minimal pendant dominating set, let us consider $S' = S - \{v_3\} = \{v_1, v_2, v_4\}$ in which the pendant vertex $\{v_7\}$ is not dominated by any of the vertices of S'. Thus S' is not a pendant dominating set.Therefore S is the minimal pendant dominating set and hence the pendant domination number of 4-sunlet graph is 4 i.e $\gamma_{pe}(S_4) = 4$ □

Theorem 3.3. *The pendant domination number of a 5-sunlet graph is* 5

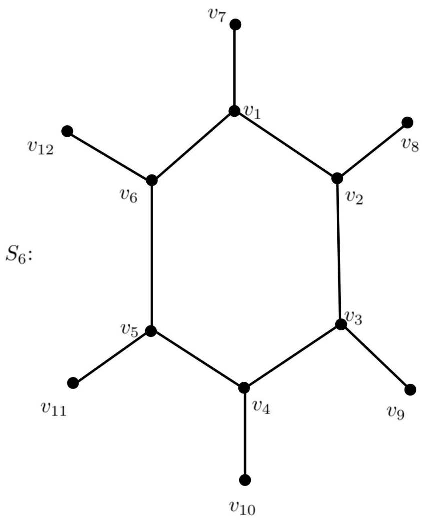
$$\gamma_{pe}(S_5) = 5$$



5 - Sunlet graph

Theorem 3.4. *The pendant domination number of a 6-sunlet graph is* 6

$$\gamma_{pe}(S_6) = 6$$

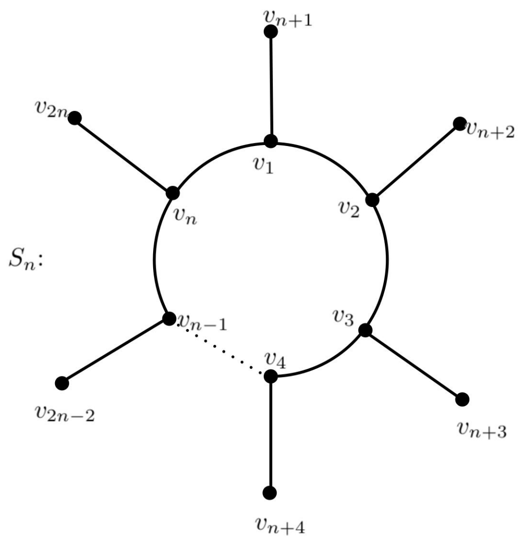


6 - Sunlet graph

Proposition 3.3. [9] *The domination number of a n-sunlet graph is n i.e.* $\gamma(S_n) = n$

Theorem 3.5. *The pendant domination number of a n-sunlet graph is n where* $n \geq 3$

$$\gamma_{pe}(S_n) = n$$



n - Sunlet graph

Proof. Let $S = \{v_1, v_2, ..., v_n\}$ be a vertex set and the vertices of a n - sunlet graph is $V(S_n) = \{v_1, v_2, ..., v_{n-1}, v_n, v_{n+1}, ..., v_{2n-1}, v_{2n}\}$ in which the set $\{v_{n+1}, v_{n+2}, ..., v_{2n}\}$ are the pendant vertices whose degree is 1. For each vertex $\{v_n\} \in S$ there exists a vertex $\{u_n\} \in V - S$ such that $\{u_n\} \cap S = \{v_n\}$ and $\{v_n\}$ is the isolated pendant vertex in S and thus S is a pendant dominating set of S_n.

To prove S is a minimal pendant dominating set, let us consider $S^{'} = S - \{v_n\} = \{v_1, v_2, ..., v_{n-1}\}$ in which the pendant vertices $\{v_{n+1}\}$ and $\{v_{n-1}\}$ are adjacent to the vertices of $S^{'}$, but $\{v_{2n+1}\}$ is not adjacent to any one vertices of $S^{'}$. Thus $S^{'}$ is not a pendant dominating set. Therefore S is the minimal pendant dominating set and hence the pendant domination number of n-sunlet graph is n i.e $\gamma_{pe}(S_n) = n$ □

4 Split and Connected pendant domination of S_n

Proposition 4.1. [9] *The split domination number of a 3-sunlet graph is* 3

$$\gamma_s(S_3) = 3$$

Theorem 4.1. *The split pendant domination number of a 3-sunlet graph is* 3

$$\gamma_{spe}(S_3) = 3$$

Proof. Let $S = \{v_1, v_2, v_3\}$ be a vertex set and $\{v_4, v_5, v_6\}$ are the pendant vertices whose degree is 1. By Theorem 3.2. we have S is the pendant dominating set of S_3 as all of the vertices of S_3 is adjacent to atleast one pendent vertex.
Let $S^{'} = S - \{v_1\} = \{v_2, v_3\}$ where the vertices $\{v_2\}$ and $\{v_3\}$ are adjacent to each other. On removal of any one vertex from S, $S^{'}$ cannot be a pendant dominating set of S_3. Now let us consider the disconnected graph $S_3{}^{'} = S_3 - S^{'} = \{v_1, v_4, v_5, v_6\} = \{v_1, v_2, v_3\}$. Here the vertices $\{v_2\}$ and $\{v_3\}$ are adjacent to each other and the vertex $\{v_1\}$ is isolated pendant vertex. The dominating set $S_3{}^{'}$ is a split pendant domination

set. The pendant dominating set $S_3^{'}$ is a split pendant dominating set as the induced subgraph $<S_3 - S>$ is disconnected. Therefore the split pendant domination number of a 3-sunlet graph is 3. i.e. $\gamma_{spe}(S_3) = 3$ □

Proposition 4.2. [9] *The split domination number of a 4-sunlet graph is* 4

$$\gamma_s(S_4) = 4$$

Theorem 4.2. *The split pendant domination number of a 4-sunlet graph is* 4

$$\gamma_{spe}(S_4) = 4$$

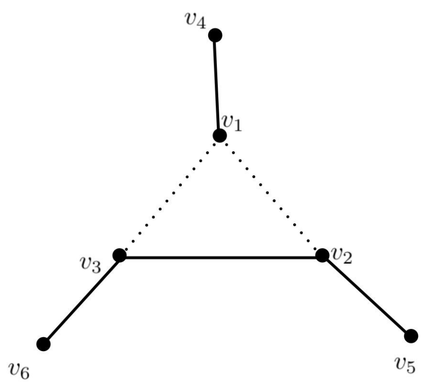
Proof. Followed by Theorem 4.2 □

Proposition 4.3. [9] *The split domination number of a n-sunlet graph is n*

$$\gamma_s(S_n) = n$$

Theorem 4.3. *The split pendant domination number of a n-sunlet graph is n where* $n \geq 3$

$$\gamma_{spe}(S_n) = n$$



3 - Split Sunlet graph

Proof. Let $S = \{v_1, v_2, ..., v_n\}$ be a vertex set and $\{v_{n+1}, v_{n+2}, ..., v_{2n}\}$ are the pendant dominating set. By Theorem 3.6. we have S is the pendant dominating set of S_n as all of the vertices of S_n as adjacent to to atleast one pendent vertex
Let us consider, $S' = S - \{v_n\}$ then the n-sunlet graph becomes disconnected graph with pendent vertex. On removal of any one vertex from S, S' cannot be a pendant dominating set of S_n. Now let us consider the disconnected graph $S_n^{'} = S_n - S'$ in which any of the two vertices are adjacent to each other and one of the vertex is an isolated pendant vertex. The dominating set $S_n^{'}$ is a split pendant domination set. The pendant dominating set $S_n^{'}$ is a split pendant dominating set as the induced subgraph $<S_n - S>$ is disconnected. Therefore the split pendant domination number of a n-sunlet graph is n. i.e. $\gamma_{spe}(S_n) = n$ □

Proposition 4.4. [9] *The connected domination number of a n-sunlet graph is n*

$$\gamma_c(S_n) = n$$

Theorem 4.4. *The connected pendant domination number of a n-sunlet graph is n*

$$\gamma_{cpe}(S_n) = n$$

Proof. Let $S = \{v_1, v_2, ..., v_n\}$ be a vertex set and $\{v_{n+1}, v_{n+2}, ..., v_{2n}\}$ are the pendant dominating set. The set S is the pendant dominating set of S_n as all of the vertices of S_n as adjacent to to atleast one pendent vertex of S. On removing any one of the vertex from S it cannot be a pendant dominating set.
Let us consider, $S^{'} = S - \{v_n\}$ there is a path between all pairs of the existing vertices of S_n except the vertex $\{v_n\}$ which makes the graph connected. Therefore S is the minimal connected pendant dominating set of S_n. The cardinality of S is the connected pendant domination number of n-sunlet graph which is n. i.e. $\gamma_{cpe}(S_n) = n$ □

Theorem 4.5. *The non-split pendant domination number of a n-sunlet graph is n*

$$\gamma_{nspe}(S_n) = n$$

Theorem 4.6. *For any n-sunlet graph we have,* $\gamma_{pe}(S_n) = \gamma_{spe}(S_n) = \gamma_{cpe}(S_n) = n$

Proof. Let $S_n = \{v_1, v_2, ..., v_{2n}\}$ be a vertex set of a n-sunlet graph. By Theorem 3.3. we have for a n-sunlet graph of n vertices the pendant domination number is n. The pendant dominating set with the minimum cardinality is n. i.e. $\gamma_{pe}(S_n) = n$.
Suppose S is the minimal pendant dominating set, on contradiction if there exists a vertex $\{v_n\} \in S$ such that the vertex $\{v_n\}$ does not satisfy any condition. Then by Theorem 4.3. we have $S^{'} = S - \{v_n\}$ is a pendant dominating set and the induced subgraph $< V - S >$ is disconnected. Thus it is a split pendant dominating set and the split pendant domination number is n. i.e.$\gamma_{spe}(S_n) = n$
As the induced subgraph $< S >$ is connected, the pendant dominating set of S of G is connected pendant dominating set and the connected pendant domination number is n. i.e. $\gamma_{cpe}(S_n) = n$ by Theorem 4.4.
Hence the result $\gamma_{pe}(S_n) = \gamma_{spe}(S_n) = \gamma_{cpe}(S_n) = n$ □

5 Conclusions

In this paper we have extended the results of domination number of n-sunlet graph S_n to pendant domination number, split pendant domination number,connected pendant domination number and non-split pendant domination number of n-sunlet graph S_n.

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